Math 330: Abstract Algebra

Final Exam Study Guide

The Final exam will be on Friday December 13 1:00-3:00.

The exam will be cumulative, though it will focus a bit more on material from the second half of the course.

A detailed list of material from the text that we have covered in the course is on the web at:

`http://www.math.uic.edu/~ marker/math330/topics.html`

Exam Week Office Hours

Wednesday	3:00–6:00
Thursday	10:00–12:00, 3:00–6:00
Friday	9:00–11:00

A large portion of the exam test that you know the important definitions and theorems, and that you can apply them to study specific examples.

The exam will also have one or two proofs, at the level of moderate homework problems.

Here is a list of some of the things you should know for the exam.

1. Know all important definitions and how to apply them: groups, subgroups, cyclic groups, order of a group, order of an element, Abelian groups, permutation groups, isomorphism, automorphism, cosets, direct product, normal subgroup, factor group, group homomorphisms, kernel, rings, units of rings, zero divisors, integral domains, fields, ideals, factor rings, prime ideals, maximal ideals, ring homomorphism, field of quotients, priciple ideal domains, irreducible elements in a ring, unique factorization domains, extension fields, splitting fields, algebraic and transcendental elements ....

2. Know how to state and use the important theorems and their corollaries: Fundamental Theorem of Cylic Groups, Caley's Theorem, Lagrange's Theorem, Cauchy's Theorem for Abelian Groups, First Isomorphism Theorem for Groups, Fundamental Theorem for Finite Abelian Groups, existence of factor rings, the characterization of prime and maximal ideals in terms of factor rings, the division algorithm for polynomial rings, the polynomial ring $F[X]$ is principle ideal domain and a unique factoriztaion domain, Fundamental Theorem of Field Theory, characterization of finite fields...

3. Understand the important examples of groups and how to work with them, such as: $\mathbb{Z}, \mathbb{R}, \mathbb{R}^*, \mathbb{C}, \mathbb{C}^*$ $\mathbb{Z}_n, U(n), D_n, S_n, A_n, GL(n,\mathbb{R}), SL(n,\mathbb{R}), GL(n,\mathbb{Z}_p)\ldots.$

Understand the important examples of rings and how to work with them for example: $\mathbb{Z}_n$, $F[X]$ for F a field, $F[X]/\langle p\rangle$ for $p \in F[X]$ irreducible.

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Sample Final Exam

Look also at the problems on the midterm and sample midterm

1) Define the following concepts:
a) H is a normal subgroup of G and K is the factor group G/H.
b) $I \subseteq R$ is a maximal ideal;
c) $a \in R$ is irreducible.
d) $\phi : R \to S$ is a ring homomorphism.

2) Decide if the following statements are TRUE or FALSE. If FALSE give a counterexample or an explanation why the statement is false.
a) If R is a commutative ring and $I \subseteq R$ is an ideal, then R/I is commutative.
b) If G is a finite Abelian group and n divides $|G|$, then G has an element of order n.
c) If F is a field, then $F[X,Y]$ is a principal ideal domain.
d) There is a field F with $|F| = 81$.
e) If G is a group, H is a subgroup of G and H is Abelian, then H is a normal subgroup of G.
f) If R is a ring and $ab = ac$, then $b = c$.
g) If F is a field, $I \subseteq F$ is an ideal, $a \in F \setminus \{0\}$ and $a \in I$, then $I = F$.

3) State the following theorem:
a) LaGrange's Theorem
b) The Fundamental Theorem of Field Theory

4) a) Find all Abelian groups (up to isomorphism) of order 16.
b) Which of these groups is isomorphic to $\mathbb{Z}_4 \times \mathbb{Z}_{24}/\langle(2,4)\rangle$?

5) a) Show that $p(X) = X^3 + X + 1$ is irreducible in $\mathbb{Z}_2[X]$.
b) Suppose F is an exension field of $\mathbb{Z}_2$, $\alpha \in F$ and $p(\alpha) = 0$. Find $a, b, c \in \mathbb{Z}_2$ such that

$$a\alpha^2 + b\alpha + c = (\alpha^2 + 1)(\alpha + 1).$$

6) Find a noncylic subgroup of order 4 in $\mathbb{Z}_4 \oplus \mathbb{Z}_{10}$.

7) Show that the homomorphic image of a principal ideal domain is a prinipal ideal domain.

8) Suppose R is a commutative ring and $I \subset R$ an ideal. Let

$$\sqrt{I} = \{a \in R : a^n \in I\}$$

for some n.
a) Show that $\sqrt{I}$ is an ideal.
b) Show that the factor ring $R/\sqrt{I}$ has no nonzero nilpotent elements (recall that $a \in R$ is *nilpotent* if $a^n = 0$ for some $n = 1, 2, \ldots$).