

Math 413 Analysis I

Bonus Problem 1

Bonus Problem 1 Let $x_1 = 1$ and let

$$x_{n+1} = 1 + \frac{1}{1 + x_n}.$$

a) Prove that $(x_n)_{n=1}^{\infty}$ converges and $\lim x_n = \sqrt{2}$. [Hint: it might be helpful to consider the sequence (x_1, x_3, x_5, \dots) and (x_2, x_4, x_6, \dots) separately.]

b) Explain how this justifies the *continued fraction* expansion

$$\sqrt{2} = 1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \dots}}}$$