Math 413 Analysis I

Bonus Problem 2

Bonus Problem 2 Recall that a sequence $(a_n)_{n=1}^{\infty}$ is *strictly monotonic* if either:

i)(a_n) is increasing i.e. $a_1 < a_2 < a_3 < \ldots$,

ii) (a_n) is decreasing i.e $a_1 > a_2 > a_3 > \ldots$, or

ii) (a_n) is constant i.e. $a_1 = a_2 = a_3 = \ldots$.

Prove that every sequence $(a_n)_{n=1}^{\infty}$ has a strictly monotonic subsequence. Use this fact to give a different proof of the Bolzano–Weierstrass Theorem.