**Math 413 Analysis I**

Bonus Problems 4 & 5

**Bonus Problem 4** If $I = (a, b)$ is an interval, then the length of $I$ is $(b - a)$. If $O = \bigcup_{n=1}^{\infty} I_n$ where $I_i \cap I_j = \emptyset$ for $i \neq j$, then the length of $O$ is the sum of the lengths of the $I_i$. If $O \subseteq [0, 1]$ has length $l$, then the closed set $[0, 1] \cap O^c$ has length $1 - l$.

Modify the construction of the Cantor set as follows. Fix $0 < \alpha \leq 1$. Such that the intervals removed at the $n^{\text{th}}$ step has length $\frac{\alpha}{3^n}$. Prove that the set $C^*$ constructed is closed, totally disconnected, nowhere dense, and has length $1 - \alpha$.

**Bonus Problem 5** For any set $A$ let $L(A)$ denote the limit points of $A$.

a) Prove that $L(A)$ is a closed set.

b) Give an example of a closed set $F$ where

$$F \supset L(F) \supset L(L(F)) \supset L(L(L(F))) = \emptyset.^{1}$$

c) Give an example of a closed set $F$ where

$$F \supset L(F) \supset L(L(F)) = L(L(L(F))) \neq \emptyset.$$

[1] Note: We write $A \subset B$ if $A \subseteq B$ but $A \neq B$.