Math 413 Analysis I
Problem Set 1

Due Friday August 29

The following problems are to be turned in neatly written, with one problem per page.

Some Notation

• If $A \subseteq \mathbb{R}$ we let

$$A^c = \mathbb{R} \setminus A = \{x \in \mathbb{R} : x \notin A\}$$

and call A^c the *complement* of A.

• Suppose I is any set and we are given sets A_i for each $i \in I$. Then

$$\bigcup_{i \in I} A_i = \{x : x \in A_i \text{ for some } i \in I\}$$

and

$$\bigcap_{i \in I} A_i = \{x : x \in A_i \text{ for every } i \in I\}.$$

1) (De Morgan's Laws) Suppose I is a set and $A_i \subseteq \mathbb{R}$ for all $i \in I$.

a) If $x \in \left(\bigcap_{i \in I} A_i\right)^c$, explain why $x \in \bigcup_{i \in I} A_i^c$. This shows $\left(\bigcap_{i \in I} A_i\right)^c \subseteq \bigcup_{i \in I} A_i^c$.

b) Prove the reverse inclusion $\left(\bigcap_{i \in I} A_i\right)^c \supseteq \bigcup_{i \in I} A_i^c$ and conclude that $\left(\bigcap_{i \in I} A_i\right)^c = \bigcup_{i \in I} A_i^c$

2) Let $y_1 = 1$ and for each $n \in \mathbb{N}$ define

$$y_{n+1} = \frac{3y_n + 4}{4}.$$

a) Use induction to prove that the sequence satisfies $y_n < 4$ for all $n \in \mathbb{N}$.

b) Use another induction argument to show that the sequence $(y_1, y_2, \ldots)$ is increasing.

3) Decide if the following statements are true or false. If true, give a proof. If false, give a counterexample.

a) If $A_1, A_2, \ldots$ are nonempty infinite subsets of $\mathbb{R}$ and $A_1 \supseteq A_2 \supseteq \ldots$, then $\bigcap_{i=1}^{\infty} A_i$ is nonempty.

b) If $A_1, A_2, \ldots$ are nonempty finite subsets of $\mathbb{R}$ and $A_1 \supseteq A_2 \supseteq \ldots$, then $\bigcap_{i=1}^{\infty} A_i$ is nonempty.