

Math 413 Analysis I
Problem Set 10

Due Friday November 7

Do the following problems from Abbott's *Understanding Analysis*.

Exercise 4.4.4

Exercise 4.4.6

Exercise 4.5.2 Give reasons for your answers.

1) Suppose $f : [-1, 1] \rightarrow \mathbb{R}$ is continuous and $x^2 + (f(x))^2 = 1$ for all $x \in [-1, 1]$. Prove that either $f(x) = \sqrt{1 - x^2}$ for all x or $f(x) = -\sqrt{1 - x^2}$ for all x . [Hint: Suppose $a, b \in (-1, 1)$, $f(a) = \sqrt{1 - a^2}$ and $f(b) = -\sqrt{1 - b^2}$.]

2) Suppose $f : \mathbb{R} \rightarrow \mathbb{R}$ is continuous, $f(x) > 0$ for all x and

$$\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow -\infty} f(x) = 0.$$

Prove that there is $a \in \mathbb{R}$ such that $f(x) \leq f(a)$ for all $x \in \mathbb{R}$.

Bonus Problem 7 Do problems 4.6.4 and 4.6.5. (Turn in by November 14)