Math 413 Analysis I
Problem Set 2

Due Friday September 5

1) Prove that if $x > -1$, then $(1+x)^n \geq 1 + nx$ for all $n \in \mathbb{N}$. [Hint: Use induction on $\mathbb{N}$.]

2) Suppose $A, B \subseteq \mathbb{R}$ are nonempty and $a < b$ for all $a \in A$ and $b \in B$.
a) Prove that $\sup A \leq b$ for all $b \in B$.
b) Prove that $\sup A \leq \inf B$.
c) Must it be true that $\sup A < \inf B$?

3) Suppose $A, B \subseteq \mathbb{R}$ are nonempty and bounded above. Let

$$A + B = \{a + b : a \in A, b \in B\}.$$

a) Prove that $A + B$ is bounded above.
b) Prove $\sup(A + B) \leq \sup A + \sup B$.
c) Prove $\sup(A + B) \geq \sup A + \sup B$ and conclude that $\sup(A + B) = \sup A + \sup B$.

4) Suppose $y_0 \neq 0$, $\epsilon > 0$ and

$$|y - y_0| < \min\left(\frac{|y_0|}{2}, \frac{\epsilon|y_0|^2}{2}\right).$$

a) Prove that $|y| > |y_0/2| > 0$.
b) Prove that

$$\left|\frac{1}{y} - \frac{1}{y_0}\right| < \epsilon.$$