**Math 413 Analysis I**
Problem Set 3

**Due Friday September 12**

Do the following problems from Abbott's *Understanding Analysis* and problems 1) and 2) below.

Exercise 2.2.1

Exercise 2.2.8

1) Prove that the sequence $(a_n)_{n=1}^{\infty}$ converges to $a$ if and only if the sequence $(a_{2n-1})_{n=1}^{\infty}$ converges to $a$ and the sequence $(a_{2n})_{n=1}^{\infty}$ converges to $a$. [Note: the first sequence is $(a_1, a_3, a_5, \ldots)$ and the second is $(a_2, a_4, a_6, \ldots)$.]

2) Recall that $A$ is *dense* if for any $a < b$ there is $c \in A$ with $a < c < b$. For example, we have shown that $\mathbb{Q}$ and $\mathbb{Q}^c$ are both dense.

We say that $A \subseteq \mathbb{R}$ is *nowhere dense* if for any $a < b$ there are $c$ and $d$ such that $a < c < d < b$ and $A \cap (c, d) = \emptyset$.

a) Give an example of a set that is infinite but nowhere dense.

b) Give an example of a set that is neither dense nor nowhere dense.

Suppose $A_1, A_2, \ldots$ are nowhere dense sets.

c) Construct a sequence of intervals $I_0 \supseteq I_1 \supseteq I_2 \ldots$ such that $I_n = [a_n, b_n]$where $a_n < b_n$ and $I_n \cap A_n = \emptyset$ for $n = 1, 2, \ldots$.

d) Use the construction from b) to conclude that $\mathbb{R} \neq \bigcup_{i=1}^{\infty} A_i$.

You have proved that $\mathbb{R}$ is not a countable union of nowhere dense sets. This is called the *Baire Category Theorem.*