Due Friday September 19

Do the following problems from Abbott’s *Understanding Analysis* and problems 1) and 2) below.

Exercise 2.3.3
Exercise 2.3.8

1) Suppose \( a_n \geq 0 \) for all \( n \).
   a) Prove that if \( \lim a_n = 0 \), then \( \lim \sqrt{a_n} = 0 \).

Suppose \( \lim a_n = a > 0 \).
   b) Prove that there is \( r > 0 \) and \( N \in \mathbb{N} \) such that \( \sqrt{a_n} + \sqrt{a} \geq r \) for all \( n \geq N \).
   c) Prove that \( \lim \sqrt{a_n} = \sqrt{a} \). Hint: Note that

\[
|\sqrt{a_n} - \sqrt{a}| = \frac{|a_n - a|}{\sqrt{a_n} + \sqrt{a}}.
\]

2) (Cesaro Means) Suppose \( \lim x_n = x \). Let \( y_n = \frac{x_1 + x_2 + \cdots + x_n}{n} \). This exercise will prove that \( \lim y_n = x \).

Fix \( \epsilon > 0 \).
   a) Prove that there is \( N_1 \) such that

\[
\sum_{i=1}^{n} \frac{|x_i - x|}{n} < \frac{\epsilon}{2}
\]

for all \( n \geq N_1 \).
   b) Prove that there is \( N_2 \) such that

\[
\sum_{i=1}^{N_1-1} \frac{|x_i - x|}{n} < \frac{\epsilon}{2}
\]

for all \( n \geq N_2 \).
   c) Finish the proof by showing that if \( N = \max(N_1, N_2) \), then \( |y_n - x| < \epsilon \) for all \( n \geq N \).