Math 413 Analysis I
Problem Set 4

Due Friday September 19

Do the following problems from Abbott's *Understanding Analysis* and problems 1)and 2) below.
Exercise 2.3.3
Exercise 2.3.8

1) Suppose $a_n \geq 0$ for all n.
a) Prove that if $\lim a_n = 0$, then $\lim \sqrt{a_n} = 0$.
Suppose $\lim a_n = a > 0$.
b) Prove that there is $r > 0$ and $N \in \mathbb{N}$ such that $\sqrt{a_n} + \sqrt{a} \geq r$ for all $n \geq N$.
c) Prove that $\lim \sqrt{a_n} = \sqrt{a}$. Hint: Note that

$$|\sqrt{a_n} - \sqrt{a}| = \frac{|a_n - a|}{\sqrt{a_n} + \sqrt{a}}.$$

2) (Cesaro Means) Suppose $\lim x_n = x$. Let $y_n = \frac{x_1 + \ldots + x_n}{n}$. This exercise will prove that $\lim y_n = x$.
Fix $\epsilon > 0$.
a) Prove that there is N_1 such that

$$\sum_{i=N_1}^{n} \frac{|x_i - x|}{n} < \frac{\epsilon}{2}$$

for all $n \geq N_1$.
b) Prove that there is N_2 such that

$$\sum_{i=1}^{N_1 - 1} \frac{|x_i - x|}{n} < \frac{\epsilon}{2}$$

for all $n \geq N_2$.
c) Finish the proof by showing that if $N = \max(N_1, N_2)$, then $|y_n - x| < \epsilon$ for all $n \geq N$.