## Math 413–Midterm 2 Study Guide

• The midterm exam will be on Wednesday November 26.

• The exam will cover sections 4.1–4.6, 5.1–5.3 . The course web page contains a week-by-week syllabus that gives a more detailed description of the material you are responsible for.

`http://www.math.uic.edu/~marker/math413/wtow.html`

• You should be comfortable with all of the key definitions. You should be able to state, apply and sketch the proof of all of the key results. The course web page contains a list of key concepts and key results.

`http://www.math.uic.edu/~marker/math413/concepts.html`

## Sample Midterm

1) Give complete definitions of the following concepts:

a) $\lim_{x \to c} f(x) = L$ where $f : A \to \mathbb{R}$;

b) $f : \mathbb{R} \to \mathbb{R}$ is uniformly continuous;

c) $f$ is differentiable at $a$;

d) $A \subseteq \mathbb{R}$ is an $F_\sigma$-set.

2) State and prove the Rolle's Theorem.

3) Determine if each of the following statements is TRUE or FALSE. If FALSE give an example showing that it is FALSE.

a) If $f$ is differentiable at $a$, then $f$ is continuous at $a$.

b) If $f$ is continuous at $a$, then $f$ is differentiable at $a$.

c) If $K$ is compact and $f : K \to \mathbb{R}$ is continuous then the image $f(K)$ is bounded.

d) If $f : \mathbb{R} \to \mathbb{R}$ is continuous and $A \subseteq \mathbb{R}$ is closed, then $f(A)$ is closed.

e) If $f : [0, 1] \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are continuous, then $g \circ f$ is uniformly continuous.

f) If $f : [a, b] \to \mathbb{R}$ is continuous, there are $c, d \in \mathbb{R}$ with $c \leq d$ such that the image $f([a, b]) = [c, d]$.

4) Suppose $f : [0, 1] \to \mathbb{R}$ and $g : [0, 1] \to \mathbb{R}$ are continuous and $f(x) > g(x)$ for all $x \in [0, 1]$. Prove that there is $a > 0$ such that $f(x) \geq g(x) + a$ for all $x \in [0, 1]$.

5) Let $f(x) = \sqrt{1 + x}$. Use the Mean Value Theorem to prove that $f(x) < 1 + \frac{x}{2}$ for all $x > 0$.