Math 414 Analysis II

Problem Set 12

Due Friday April 30

1) Consider the initial value problem $g'(x) = g(x)$ where $g(0) = 1$. Let $g_0(x) = 1$ be our initial approximation to a solution and follow the proof the existence of solutions for ordinary differential equations to find approximate solutions $g_0, g_1, \ldots$ and prove that they converge to e^x.

2) Let $f(x,y) = x^2 + y^2 - 1$. Since $f(0,1) = 0$ and $\frac{\partial f}{\partial y}(0,1) = 2$, there is a continuous function g defined on a neighborhood of 0 such that $f(x, g(x)) = 0$ and $g(0) = 1$. We can use the proof of the Implicit Function Theorem to find a sequence of fuctions (g_n) converging to g with $g_0(x) = 1$. Find the first four nonzero terms of the Talyor series for $g(x)$.

In this case we can solve directly and get that $g(x) = \sqrt{1 - x^2}$. Use Binomial Series to find the Taylor series for $g(x)$ and show that it agrees with your calculation on the first four nonzero terms.