Math 414 Analysis II

Problem Set 5

Due Friday February 20

1) Suppose $A \subseteq \mathbb{R}$ has measure zero.

a) Let $B = \{x \in [0,1] : x \notin A\}$. Prove that B is dense and does not have measure zero.

b) Fix $r \in \mathbb{R}$ and let $A + r = \{x + r : x \in A\}$. Prove that $A + r$ has measure zero.

c) Prove or Disprove. Let C be the Cantor set. If $x \in [0,1]$ there is $y \in C$ and $z \in \mathbb{Q}$ such that $x = y + z$. [Hint: use b)]

2) Suppose $f : [a,b] \to \mathbb{R}$ is integrable, $f(x) \geq 0$ for all x and $\int_a^b f = 0$. Prove that $\{x : f(x) \neq 0\}$ has measure zero.

3) Suppose $f : [a,b] \to \mathbb{R}$. For $n \in \mathbb{N}$ let $h = \frac{b-a}{n}$, let $x_i = a + ih$ for $i = 0, \ldots, n$ and let

$$s_n = \sum_{i=1}^{n} f(x_{i-1})h.$$

a) Prove, using results from class, that if f is integrable then $\lim s_n = \int_a^b f$.

b) Give an example of a nonintegrable function f where $\lim s_n$ exists.[1]

[1] This example shows why we don't only use partitions where every interval has the same length.