**Math 414–Midterm 1 Study Guide**

• The midterm exam will be on Wednesday November 26.

• The exam will cover sections 7.2–7.6, 8.1, 2.7, and2.8 . The course web page contains a week-by-week syllabus that gives a more detailed description of the material you are responsible for.

`http://www.math.uic.edu/~marker/math414/wtow.html`

• You should be comfortable with all of the key definitions. You should be able to state, apply and sketch the proof of all of the key results. The course web page contains a list of key concepts and key results.

`http://www.math.uic.edu/~marker/math414/concepts.html`

**Sample Midterm**

1) a) Let $f : [a, b] \to \mathbb{R}$ be a bouned function. Give a complete definition of what it means for $f$ to be integrable and $\int_a^b f = L$, (including the definition of upper and lower sums).

b) Define the following concepts: $A \subseteq \mathbb{R}$ has measure zero, $\sum_{n=1}^{\infty} a_n$ converges.

2) State Lebesgue's Criteria for integrability.

3) State and Prove the Ratio Test for convergence.

4) Decide if the following statements are TRUE or FALSE. If FALSE, prove justify your answer.

a) If $f : [a, b] \to \mathbb{R}$ is differentiable, then $f'$ is integrable.

b) If $\{x \in [a, b] : f(x) > 0\}$ is infinite, $f$ is integrable and $f \geq 0$ for all $x \in [a, b]$, then $\int_a^b f > 0$

c) The series $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2 + 1}$ converges.

d) The series $\displaystyle\sum_{n=1}^{\infty} \frac{10^n}{n!}$ converges.

e) Every differentiable $f : [a, b] \to \mathbb{R}$ is integrable.

5) Suppose $(a_n)$ and $(b_n)$ are sequences with $a_n, b_n > 0$ such that $\lim \frac{a_n}{b_n} = 1$. Prove that $\sum a_n$ converges if and only if $\sum b_n$ converges.

6) Suppose $f : [a, b] \to \mathbb{R}$ is integrable, $f(x) \geq 0$ for all $x$, $f$ is continuous at $c$ for some $a < c < b$ and $f(c) > 0$. Prove that $\int_a^b f > 0$.