

Math 435 Number Theory I
Problem Set 5

Due: Friday October 7:

1) Solve the following system of congruences.

$$x \equiv 1 \pmod{24}, x \equiv 13 \pmod{18}, x \equiv 17 \pmod{20} \text{ and } x \equiv 37 \pmod{75}.$$

2) a) What is the remainder when 6^{2003} is divided by 11?

b) What is the smallest positive residue of $2^{1000000} \pmod{7}$.

3) Find all solutions to

$$x^2 + 6x - 31 \equiv 0 \pmod{72}$$

[Hint: First find all solutions mod 8 and mod 9].

4) Prove that

$$\frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}$$

is an integer for all $n \in \mathbb{Z}$.