Math 435 Number Theory I
Problem Set 8

Due: Friday October 28:

1) We call n a *Carmichael Number* if n is not prime but $a^n \equiv a \pmod{n}$ for all a. Prove that 1105 is a Carmichael Number.

2) We say that a function f is *multiplicative* if $f(nm) = f(n)f(m)$ for all n, m with $\gcd(n, m) = 1$.

a) Let $\lambda(n) = (-1)^{k_1+\ldots+k_s}$ where $n = p_1^{k_1} \cdots p_m^{k_s}$ with $p_1, \ldots, p_s$ distinct primes. Prove that λ is multiplicative.

b) Suppose f is multiplicative. Define $F(n) = \sum_{d|n} f(d)$. Prove that F is multiplicative. [Hint: First show that if Let $\gcd(m, n) = 1$, then $(a, b) \mapsto ab$ is a one-to-one onto map between $\{(a, b) : a|n, b|m\}$ and $\{d : d|mn\}$.]

3) Solve $x^{25} \equiv 2 \pmod{437}$

4) (5pt Bonus) We say that $n \in \mathbb{N}$ is *square free* if there is no prime p such that $p^2|m$. Prove that if n is square free and $\gcd(k, n) = 1$, then $x \mapsto x^k \pmod{n}$ is a one-to-one onto map from $\mathbb{Z}_n$ to $\mathbb{Z}_n$.