

Math 502 Metamathematics I
Problem Set 3

Due: Monday October 4

1) Prove that $\phi \vdash \neg\neg\phi$. [Hint: You might want to start by observing that $\neg\neg\neg\phi \vdash \neg\phi$ as in the example on page 16.]

2) Show that the following version of the contraposition inference rule is derivable.

$$\frac{\Gamma, \neg\phi \vdash \psi}{\Gamma, \neg\psi \vdash \phi}$$

3) Show that the following two inference rules using \wedge are derivable. [Hint: You will need to use that $(\phi \wedge \psi)$ is an abbreviation for $\neg(\neg\phi \vee \neg\psi)$.]

a)

$$\frac{\Gamma \vdash (\phi \wedge \psi)}{\Gamma \vdash \phi}$$

b)[†]

$$\frac{\Gamma \vdash \phi \quad \Gamma \vdash \psi}{\Gamma \vdash (\phi \wedge \psi)}$$

4) Show that the following inference rule is derivable. [Hint: You will need that $\forall x \phi$ is an abbreviation for $\neg\exists x \neg\phi$. As a strategy, you might try to derive a contradiction from $\Gamma, \neg\phi$.]

$$\frac{\Gamma \vdash \forall x \phi}{\Gamma \vdash \phi}$$