**Math 502 Metamathematics I**
Problem Set 6

**Due: Wednesday December 10**:
Do 4 of the following problems.

1) Fill in the details of the proof of Church's Theorem. We need to show that $\theta_n^P$ is valid if and only if $P$ halts on input $n$.

2)$^\dagger$ Suppose $f : \mathbb{N} \to \mathbb{N}$ is RM-computable. Prove that there is a register machine program $P$ computing $f$ using only 3 registers. [Hint: We will have to use these registers to code a simulation of the computation of $P$.]

3) a) (Reduction) Suppose $A$ and $B$ are recursively enumerable. Show that are are recursively enumerable sets $A_0 \subseteq A$ and $B_0 \subseteq B$ such that $A_0 \cap B_0 = \emptyset$ and $A_0 \cup B_0 = A \cup B$.

b) (Separation) Suppose $A$ and $B$ are $\Pi_1$ and $A \cap B = \emptyset$. Show that there is a recursive $C$ such that $A \subseteq C$ and $B \cap C = \emptyset$. (HINT: Apply a) to $\mathbb{N} \setminus A$ and $\mathbb{N} \setminus B$.)

4) a) Show that $\{e : W_e \neq \emptyset\}$ is $\Sigma_1$-complete.

b) Let $Cof = \{e : \mathbb{N} \setminus W_e \text{ is finite}\}$. Show that $Cof$ is $\Sigma_3$.

c)$^\dagger$ Show that $Cof$ is $\Sigma_3$-complete.

5) Suppose $X \subset \mathbb{N}^2$ is $\Delta_n$. Show that there is a $\Delta_n$ set $Y \subseteq \mathbb{N}$ such that for all $n$, $Y \neq X_n$ (where $X_n = \{m : (n, m) \in X\}$). Thus there is no universal $\Delta_n$ set. (Hint: Consider $\{n : (n, n) \notin X\}$).