Math 502 Metamathematic I
Problem Set 1

Due: Friday September 9

Let $\mathcal{L}$ be a language.

1) Prove that every $\mathcal{L}$-formula has the same number of left and right parenthesies [ie. the same number of "(" as ")"].

2) Let $\mathcal{M}$ and $\mathcal{N}$ be $\mathcal{L}$-structures. Suppose $j : \mathcal{M} \to \mathcal{N}$ is an $\mathcal{L}$-embedding and $\sigma : V \to M$ is an assignment.

a) Prove that $j(t^{\mathcal{M}}[\sigma]) = t^{\mathcal{N}}[j \circ \sigma]$, for all $\mathcal{L}$-terms t.

b) Prove that $\mathcal{M} \models_\sigma \phi$ if and only if $\mathcal{N} \models_{j \circ \sigma} \phi$ for all quantifier free $\mathcal{L}$-formulas ϕ.

3) Let $\mathcal{M}$ be an $\mathcal{L}$-structure and let $\sigma : V \to M$ be an assignment.

a) Suppose $\mathcal{M} \models_\sigma \exists v_1 \forall v_2\ \phi$. Prove that $\mathcal{M} \models_\sigma \forall v_2 \exists v_1\ \phi$. Give an example showing that the converse is false.

b) Prove that $\mathcal{M} \models_\sigma \forall\ v_i \phi$ if and only if $\mathcal{M} \models_\sigma \neg \exists v_i\ \neg \phi$.