Math 502 Metamathematics I
Problem Set 3

Due: Wednesday October 5

1) Prove that $\phi \vdash \neg\neg\phi$.[**Hint**: You might want to start by observing that $\neg\neg\neg\phi \vdash \neg\phi$ as in the example on page 16.]

2) Show that the following version of the contrapostion inference rule is derivable.

$$\frac{\Gamma, \neg\phi \vdash \psi}{\Gamma, \neg\psi \vdash \phi}$$

3) Show that the following two inference rules using $\wedge$ are derivable. [Hint: You will need to use that $(\phi \wedge \psi)$ is an abbreviation for $\neg(\neg\phi \vee \neg\psi)$.]

a)

$$\frac{\Gamma \vdash (\phi \wedge \psi)}{\Gamma \vdash \phi}$$

b)

$$\frac{\Gamma \vdash \phi \qquad \Gamma \vdash \psi}{\Gamma \vdash (\phi \wedge \psi)}$$

4) Show that the following inference rule is derivable. [Hint: You will need that $\forall x\ \phi$ is an abreviation for $\neg\exists x\ \neg\phi$. As a strategy, you might try to derive a contradiction from $\Gamma, \neg\phi$.]

$$\frac{\Gamma \vdash \forall x\ \phi}{\Gamma \vdash \phi}$$