Due: Wednesday October 5

1) Prove that $\phi \vdash \neg \neg \phi$. [Hint: You might want to start by observing that $\neg \neg \neg \phi \vdash \neg \phi$ as in the example on page 16.]

2) Show that the following version of the contraposition inference rule is derivable.

   \[
   \frac{\Gamma, \neg \phi \vdash \psi}{\Gamma, \neg \psi \vdash \phi}
   \]

3) Show that the following two inference rules using $\land$ are derivable. [Hint: You will need to use that $(\phi \land \psi)$ is an abbreviation for $\neg (\neg \phi \lor \neg \psi)$.

   a) \[
   \frac{\Gamma \vdash (\phi \land \psi)}{\Gamma \vdash \phi}
   \]

   b) \[
   \frac{\Gamma \vdash \phi \quad \Gamma \vdash \psi}{\Gamma \vdash (\phi \land \psi)}
   \]

4) Show that the following inference rule is derivable. [Hint: You will need that $\forall x \phi$ is an abbreviation for $\neg \exists x \neg \phi$. As a strategy, you might try to derive a contradiction from $\Gamma, \neg \phi$.]

   \[
   \frac{\Gamma \vdash \forall x \phi}{\Gamma \vdash \phi}
   \]