1) a) Write register machine program to compute \( \max(x, y) \).
    b) Give primitive recursive functions \( j(x, y, s) \) and \( g_i(x, y, s) \) such that if the program from a) is given input \( x \) and \( y \), \( j(x, y, s) \) is the next instruction and \( g_i(x, y, s) \) is the contents of register \( i \) at time \( s \).

2) Write a register machine program to compute

\[
f(x, y) = \begin{cases} 
0 & \text{if } y = 0 \\
\lfloor x/y \rfloor & \text{if } y \neq 0 
\end{cases}
\]

where \( \lfloor r \rfloor \) is the greatest integer \( \leq r \) for \( r \in \mathbb{R} \).

3) Prove that \( \max(x, y) \) and \( \text{lcm}(x, y) \) are primitive recursive, where \( \text{lcm}(x, y) \) is the least common multiple of \( x, y \).

4) a) Suppose \( P(\bar{x}, y) \) is a primitive recursive predicate and \( g(\bar{x}) \) is a primitive recursive function. Define \( f(\bar{x}) = 0 \) if there is no \( n \leq g(\bar{x}) \) such that \( P(\bar{x}, n) = 1 \). Otherwise \( f(\bar{x}) \) is the least \( n \leq g(\bar{x}) \) such that \( P(\bar{x}, n) = 1 \). Prove that \( f \) is primitive recursive.
    b) Prove that the function \( f \) from problem 2 is primitive recursive.