Math 502 Metamathematics I

Problem Set 5

Due: Wednesday November 2

1) a) Write register machine program to compute $\max(x, y)$.

b) Give primitive recursive functions $j(x, y, s)$ and $g_i(x, y, s)$ such that if the program from a) is given input x and y, $j(x, y, s)$ is the next instruction and $g_i(x, y, s)$ is the contents of register i at time s.

2) Write a register machine program to compute

$$f(x,y) = \begin{cases} 0 & \text{if } y = 0 \\ \lfloor x/y \rfloor & \text{if } y \neq 0 \end{cases}$$

where $\lfloor r \rfloor$ is the greatest integer $\leq r$ for $r \in \mathbb{R}$.

3) Prove that $\max(x, y)$ and $\operatorname{lcm}(x, y)$ are primitive recursive, where $\operatorname{lcm}(x, y)$ is the least common multiple of x, y.

4) a) Suppose $P(\bar{x}, y)$ is a primitive recursive predicate and $g(\bar{x})$ is a primitive recursive function. Define $f(\bar{x}) = 0$ if there is no $n \leq g(\bar{x})$ such that $P(\bar{x}, n) = 1$. Otherwise $f(\bar{x})$ is the least $n \leq g(\bar{x})$ such that $P(\bar{x}, n) = 1$. Prove that f is primitive recursive.

b) Prove that the function f from problem 2 is primitive recursive.