

Math 502 Metamathematics I

Problem Set 6

Due: Wednesday November 16 Two more problems using the Compactness Theorem.

1) Let $\mathcal{L} = \{+, \cdot, <, 0, 1\}$. Prove there is $\mathcal{M} \models \text{Th}(\mathbb{N})$ with $a_1, a_2, \dots \in M$ such that $a_1 >^{\mathcal{M}} a_2 >^{\mathcal{M}} a_3 > \dots$

2) Let $\mathcal{L} = \{P_n : n = 2, 3, 4, 5, \dots\}$. Let \mathcal{M} be the \mathcal{L} -structure with universe \mathbb{N} such that $P_n^{\mathcal{M}} = \{m \in \mathbb{N} : m \text{ is divisible by } n\}$. Prove that there is $\mathcal{N} \models \text{Th}(\mathcal{M})$ with $a \in M$ such that $\mathcal{M} \models P_n(a)$ for $n = 2, 3, \dots$