Math 502 Metamathematics I
Problem Set 7

Due: Wednesday December 7:

1) Suppose $A \subseteq \mathbb{N}$ is recursively enumerable and infinite. Prove that there is a total recursive $f : \mathbb{N} \to \mathbb{N}$ with $\mathrm{img}(f) = A$.

2) a) (Reduction) Suppose A and B are recursively enumerable. Show that are are recursively enumerable sets $A_0 \subseteq A$ and $B_0 \subseteq B$ such that $A_0 \cap B_0 = \emptyset$ and $A_0 \cup B_0 = A \cup B$.

b) (Separation) Suppose A and B are Π_1 and $A \cap B = \emptyset$. Show that there is a recursive C such that $A \subseteq C$ and $B \cap C = \emptyset$. (HINT: Apply a) to $\mathbb{N} \setminus A$ and $\mathbb{N} \setminus B$.)

3) Show that $\{e : W_e \neq \emptyset\}$ is Σ_1-complete.

4) Let Cof$= \{e : \mathbb{N} \setminus W_e \text{ is finite}\}$. Show that Cof is Σ_3.