Due: Wednesday December 7:

1) Suppose $A \subseteq \mathbb{N}$ is recursively enumerable and infinite. Prove that there is a total recursive $f : \mathbb{N} \rightarrow \mathbb{N}$ with $\text{img}(f) = A$.

2) a) (Reduction) Suppose $A$ and $B$ are recursively enumerable. Show that there are recursively enumerable sets $A_0 \subseteq A$ and $B_0 \subseteq B$ such that $A_0 \cap B_0 = \emptyset$ and $A_0 \cup B_0 = A \cup B$.

   b) (Separation) Suppose $A$ and $B$ are $\Pi_1$ and $A \cap B = \emptyset$. Show that there is a recursive $C$ such that $A \subseteq C$ and $B \cap C = \emptyset$. (HINT: Apply a) to $\mathbb{N} \setminus A$ and $\mathbb{N} \setminus B$.)

3) Show that $\{ e : W_e \neq \emptyset \}$ is $\Sigma_1$-complete.

4) Let $\text{Cof} = \{ e : \mathbb{N} \setminus W_e \text{ is finite} \}$. Show that $\text{Cof}$ is $\Sigma_3$. 