Math 502 Metamathematics I
Problem Set 1

Due: Friday September 4

Let $\mathcal{L}$ be a language with relation symbols $\mathcal{R}$, function symbols $\mathcal{F}$ and constant symbols $\mathcal{C}$.

1) Let $T_0 = \mathcal{C} \cup \{v_1, v_2, \ldots\}$ be the variables and constants of the language. Given T_k let

$$T_{k+1} = \{f(t_1, \ldots, t_{n_f}) : f \in \mathcal{F}, t_1, \ldots, t_{n_f} \in T_k\} \cup T_k.$$

Let $T = \bigcup_{k=0}^{\infty} T_k$. Prove that T is the set of all terms. [Note: We can give a similar construction of the set of all formulas]

2) a) Prove that for every term the number of left and right parentheses are equal.

b) Prove that for every formula the number of left and right parentheses are equal.

3) Let $\mathcal{M}$ be an $\mathcal{L}$-structure and let $\sigma : V \to M$ be an assignment.

a) Suppose $\mathcal{M} \models_\sigma \exists v_1 \forall v_2\ \phi$. Prove that $\mathcal{M} \models_\sigma \forall v_2 \exists v_1\ \phi$. Give an example showing that the converse is false.

b) Prove that $\mathcal{M} \models_\sigma \forall\ v_i \phi$ if and only if $\mathcal{M} \models_\sigma \neg \exists v_i\ \neg \phi$.

c) Give an example of $\mathcal{L}$, $\mathcal{M}$, σ, ϕ and ψ such that $\mathcal{M} \models_\sigma (\exists v\ \phi \wedge \psi)$ but $\mathcal{M} \not\models_\sigma \exists v\ (\phi \wedge \psi)$.