Due Friday February 1

1) Suppose $X$ is a transitive set. Prove that $(X, \epsilon)$ is a model of the axiom of extensionality.

2) Let $C$ be a set of ordinals and let $\beta = \bigcup_{\alpha \in C} \alpha$.
   a) Prove that $\beta$ is an ordinal.
   b) Prove that $\alpha \leq \beta$ for all $\alpha \in C$.
   c) Suppose $\gamma$ is an ordinal such that $\alpha \leq \gamma$ for all $\alpha \in C$. Prove that $\beta \leq \gamma$.

   We have shown that $\beta$ is the least upper bound for the set $C$. We write $\beta = \sup C$.

3) For ordinals $\alpha$ and $\beta$ we inductively define $\alpha + \beta$ as follows.
   i) $\alpha + 0 = \alpha$;
   ii) $\alpha + (\beta + 1) = (\alpha + \beta) + 1$;
   iii) $\alpha + \beta = \sup\{\alpha + \gamma : \gamma < \beta\}$ for $\beta$ a limit ordinal.
   a) Is $1 + \omega = \omega + 1$?
   b) Prove that $(\alpha + \beta) + \gamma = \alpha + (\beta + \gamma)$ for all ordinals $\alpha, \beta, \gamma$.
   c) Let $\alpha$ and $\beta$ be ordinals. Let $X = (\{0\} \times \alpha) \cup (\{1\} \times \beta)$ and order $X$ lexicographically, i.e.,

   $$(m, x) < (n, y) \iff m < n \text{ or } m = n \text{ and } x < y.$$ 

   Prove that $X$ has order type $\alpha + \beta$. 
