Due Friday April 4

1) a) Suppose $L \models \kappa$ is a cardinal. Show that $L_\kappa \models$ comprehension.

   b) Suppose $L \models \kappa$ is a regular cardinal. Then $L_\kappa \models$ Replacement.

   c) Suppose $L \models \kappa$ is a limit cardinal. Then $L_\kappa \models$ Power Set.

2) Assume $V = L$. Let $X \prec L_{\omega_1}$.

   a) Show $\omega \in X$.

   Fix $x \in X$.

   b) Show that there is $f \in L_{\omega_1}$ such that $f : \omega \to x$ is onto.

   c) Let $f : \omega \to x$ be $<_L$-least in $L_{\omega_1}$ such that $f$ is surjective. Argue that $f \in X$ and conclude that $x \subseteq X$.

   d) Conclude that $X$ is transitive and must be equal to $L_\alpha$ for some $\alpha \leq \omega_1$.

   [Hint: Recall that the Mostowski Collapse is the identity on transitive sets.]

**Note:** The Condensation Lemma tells us that $X \cong L_\beta$ for some $\beta \leq \omega_1$. But we have, in this case, proved the much stronger result that $X$ is already some $L_\beta$. 

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