Math 506 Model Theory I
Problem Set 7

Due: Wednesday April 20

Do problem 1) below and the following problems from the text:

4.5.16 k) (you’ve already done earlier parts. In this problem and then next it might simplify to note that in the definition of $\kappa$-stability we need only look at one-types.)

4.5.22

4.5.30

4.5.45 (Assume that $T$ has infinite models.)

1) Given an $L$-structure $M$ and $A \subseteq M$ we say that $b$ is algebraic over $A$ in $M$, if there is a formula $\phi(v, \overline{a})$ with parameters $\overline{a} \in A$. Such that $M \models \phi(b, \overline{a})$ and $\{ c \in M : M \models \phi(c, \overline{a}) \}$ is finite, i.e., there is a finite $A$-definable set $X \subseteq M$ such that $b \in A$.

a) Show that if $A \subseteq M$ and $M \prec N$, then $b \in N$ is algebraic over $A$ in $N$ if and only if $b \in M$ and $b$ is algebraic over $A$ in $M$.

We let $acl(A)$ denote the elements algebraic over $A$. By a) we see that this does not depend on the ambient model $M$.

b) Prove the following easy properties of algebraic closure

i) $acl(A) \supseteq A$;

ii) If $A \subseteq B$, then $acl(A) \subseteq acl(B)$;

iii) $acl(acl(A)) = acl(A)$;

iv) If $b \in acl(A)$, then there is $A_0 \subseteq A$ finite such that $b \in acl(A_0)$.

c) Suppose $M$ is saturated, $A \subseteq M$ and $|A| < |M|$. Prove that $b \in acl(A)$ if and only if there is a finite set $B$ such that $\sigma(b) \in B$ for any automorphism $\sigma$ of $M$ such that $\sigma|A$ is the identity.

Do one at least one of d), e) , f)

d) Show that if $M \models DLO$ and $A \subseteq M$, then $acl(A) = A$.

e) Show that if $M \models ACF$ and $A \subseteq M$, then $acl(A)$ is the algebraic closure of subfield generated by $A$.

f) Let $G$ be a torsion free divisible abelian group and let $A \subseteq G$. Show that if $A \neq \emptyset$, then

$$acl(A) = \{ m_1a_1 + \ldots + m_na_n : m_i \in \mathbb{Q}, a_i \in A, n = 0, \ldots \}.$$