MTHT 430 Analysis for Teachers
Problem Set 10

Do problem 2 from Chapter 5 of Spivak's *Calculus* [You may use all the algebraic properites of limits and the continuity of $\sqrt{x}$ and $\sqrt{1-x^2}$]

1) Suppose $f : \mathbb{R} \to \mathbb{R}$, $g : \mathbb{R} \to \mathbb{R}$, $|f(x)| \le M$ for all $x \in \mathbb{R}$ and $\lim\limits_{x \to a} g(x) = 0$.
Prove that $\lim\limits_{x \to a} f(x)g(x) = 0$.

2) Suppose $f(x) \le g(x) \le h(x)$ for all $x \in \mathbb{R}$ and

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L.$$

Prove that $\lim\limits_{x \to a} g(x) = L$.

3) Suppose $|f(x)| \le |x|$ for all $x \in \mathbb{R}$. Prove that f is continuous at 0.

4) Give an example of a function f that is nowhere continuous, but $|f|$ is continuous everywhere.