MTHT 430 Analysis for Teachers
Problem Set 5–**REVISED**

1) Do Chapter 12: 1 i),ii),iii),viii) **DO NOT DO THIS PROBLEM. THIS WILL BE ON THE NEXT PROBLEM SET**

2) Suppose X and Y are sets and $f : X \to Y$. Recall that if $A \subseteq X$, then the *image* of A is

$$f(A) = \{f(a) : a \in A\}$$

and if $B \subseteq Y$, then the *inverse image* of B is

$$f^{-1}(B) = \{x \in X : f(x) \in B\}.$$

Recall also that $X - A = \{x \in X : x \notin A\}$.

Decide if each of the following is TRUE or FALSE. If TRUE give a proof, if FALSE give a counterexample showing that it is false.

a) If $A \subseteq X$, then $f(X - A) = f(X) - f(A)$.

b) If $A \subseteq Y$, then $f^{-1}(Y - A) = f^{-1}(Y) - f^{-1}(A)$.

c) If $A \subseteq Y$, then $f(f^{-1}(A)) = A$.

d) If $A \subseteq X$, then $f^{-1}(f(A)) = A$.

3) Suppose $f : X \to Y$ and $g : Y \to Z$. Decide if each of the following is TRUE or FALSE. If TRUE give a proof, if FALSE give a counterexample showing that it is false.

a) If f and g are one-to-one, then $g \circ f$ is one-to-one.

b) If $g \circ f$ is one-to-one, then f is one-to-one.

c) If $g \circ f$ is one-to-one, then g is one-to-one.

4) a) Prove that if $f : \mathbb{R} \to \mathbb{R}$ is increasing and $g : \mathbb{R} \to \mathbb{R}$ is increasing, then $g \circ f$ is increasing.

b) Give an example of decreasing functions f and g such that $g \circ f$ is increasing.