MTHT 430 Analysis for Teachers
Problem Set 7

1) Suppose $A, B \subseteq \mathbb{R}$ are nonempty and bounded above. Prove the following.

a) If $\sup A > c$, then there is $a \in A$ with $a > c$.

b) If $\sup A < \sup B$, there is $b \in B$ an upper bound for A.

c) For all $\epsilon > 0$ there is $a \in A$ such that $\sup A - \epsilon < a \leq \sup A$

2) Suppose $A, B \subseteq \mathbb{R}$ are nonempty and bounded above. Let

$$A + B = \{a + b : a \in A, b \in B\}.$$

a) Prove that $\sup A + \sup B$ is an upper bound for $A + B$.

b) Prove that $\sup(A + B) \leq \sup A + \sup B$.

c) Prove that $\sup(A + B) \geq \sup A + \sup B$ and conclude that

$$\sup(A + B) = \sup A + \sup B.$$

[Hint: Suppose for contradiction that $\sup A + \sup B > \sup(A + B)$. As a first step use 1a) to show there is $a \in A$ with $a + \sup B > \sup(A + B)$.]

3) Verify the following limits **using the definition** of convergence.

a) $\lim\limits_{n\to\infty} \dfrac{1}{6n^2 + 1} = 0$

b) $\lim\limits_{n\to\infty} \dfrac{3n + 1}{2n + 5} = \dfrac{3}{2}$

c) $\lim\limits_{n\to\infty} \dfrac{2}{\sqrt{n + 3}} = 0.$

4) Consider the sequence

$$0, 1, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, \ldots$$

where each string of zeros has one more zero than the previous. Does this sequence converge or diverge? If it converges to a limit L, prove that it converges to L. If it diverges, prove that it diverges.