

MTHT 430 Analysis for Teachers
Problem Set 9

- 1) **Use the definition** of limits to verify the following limits.
- a) Let f be the constant function $f(x) = \pi$. Prove that $\lim_{x \rightarrow 0} f(x) = \pi$.
 - b) $\lim_{x \rightarrow 2} x^3 = 8$.
 - c) $\lim_{x \rightarrow 2} \frac{1}{x} = \frac{1}{2}$
 - d) $\lim_{x \rightarrow 2} x^3 + \frac{1}{x} = \frac{17}{2}$ [Hint: Use a) and b)]
 - e) $\lim_{x \rightarrow 2} \frac{1}{x^2} = \frac{1}{4}$ [Hint:

$$\left| \frac{1}{x^2} - \frac{1}{a^2} \right| = |x - a| \left| \frac{1}{xa^2} + \frac{1}{x^2a} \right|.$$

f) $\lim_{x \rightarrow 3} \sqrt{2x + 3} = 3$.

- 2) We say that $\lim_{x \rightarrow a} f(x) = +\infty$ if for all $M > 0$ there is $\delta > 0$ such that $f(x) > M$ for all x with $0 < |x - a| < \delta$.

- a) Prove that $\lim_{x \rightarrow 1} \frac{1}{(x - 1)^2} = +\infty$.
- b) Does $\lim_{x \rightarrow 1} \frac{1}{x - 1} = +\infty$?
- c) Suppose $\lim_{x \rightarrow a} g(x) = 0$, $N > 0$ and $f(x) > N$ for all x . Prove that $\lim_{x \rightarrow a} \frac{f(x)}{|g(x)|} = +\infty$.