MTHT 430 Analysis for Teachers

Quiz 1–3 Solutions

Q1) Prove that if $x < y$ and $z < 0$, then $xz > xy$.

Since $x < y$ and $z < 0$, we know that $y - x > 0$ and $-z > 0$. The product of positive number is positive. Thus

$$0 < -z(y - x) = -yz + xz$$

and $xz > yz$.

Q2) Prove that $1 + 3 + 5 + \ldots + (2n - 1) = n^2$ for all $n \in \mathbb{N}$. In other words, prove that

$$\sum_{i=1}^{n}(2i - 1) = n^2.$$

We prove this by induction. First if $n = 1$, then $1 = 1^2$ so the claim is true. For purposes of induction assume the claim is true for $n = k$. Then

$$\begin{aligned} 1 + 3 + 5 + \ldots + (2(k + 1) - 1) &= (1 + 3 + \ldots + 2k - 1) + (2k + 1) \\ &= k^2 + 2k + 1 \text{ since the claim is true for } k \\ &= (k + 1)^2 \end{aligned}$$

Thus, by induction, the claim is true for all $n \in \mathbb{N}$.

Q3) Prove that if $f : X \to Y$ and $g : Y \to Z$ are onto, then $g \circ f$ is onto.

Recall: *A function $h : A \to B$ is onto if for all $b \in B$ there is $a \in A$ with $h(a) = b$. The quantifiers "for all" and "there is" are* **crucial** *here.*

Let $z \in Z$. We must show there is $x \in X$ with $g(f(x)) = z$. Since g is onto, there is $y \in Y$ with $g(y) = z$. Since f is onto, there is $x \in X$ with $f(x) = y$. Thus

$$g(f(x)) = g(y) = z$$

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