

MTHT 430 Analysis for Teachers
Midterm 1 Study Guide

- 1) Know all important definitions and how to apply them:
absolute value, binomial coefficient $\binom{n}{i}$, functions, composition of functions, one-to-one and onto functions, inverse functions, increasing and decreasing functions, images and inverse images, upper(lower bounds), sup and inf, the Completeness Axiom
- 2) Know the main results and techniques, how to prove them and how to apply them:
triangle inequality, $\binom{n+1}{i} = \binom{n}{i-1} + \binom{n}{i}$, binomial theorem, one-to-one onto functions have inverses, Archimedian Property, Density of \mathbb{Q} in \mathbb{R} , Existence of Square Roots, Nested Interval Property.
- 3) Know how to do proofs by induction, proofs by contradiction, proofs that two sets are equal, proofs of “if-then” and “if and only if” statements.

Sample Exam

The following are the style of questions I am likely to ask on the exam.

- 1) Define the following concepts
 - a) $f : X \rightarrow Y$ is one-to-one
 - b) α is the least upper bound for A
 - c) $f(A)$, the image of A under f .
 - d) $\binom{n}{i}$
- 2)
 - a) State the Completeness Axiom
 - b) State the Archimedian Property of the Real Numbers
- 3) State and prove the Triangle Inequality.

4) Decide if the following statements are TRUE or FALSE. If FALSE, give an example showing the statement is FALSE.

- a) If $f : \mathbb{N} \rightarrow \mathbb{N}$ is one-to-one, then f is onto.
- b) If $f : \mathbb{R} \rightarrow \mathbb{R}$ is one-to-one and onto, then there is an inverse function f^{-1} .
- c) If $A \subset \mathbb{R}$ is bounded above, then there is $a \in A$ a least upper bound for A .
- d) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : [0, +\infty) \rightarrow \mathbb{R}$ be the functions $f(x) = x^2$ and $g(x) = \sqrt{x}$. Then $(f \circ g)(x) = x$ and $(g \circ f)(x) = x$.
- e) If $x^2 < y^2$, then $x < y$.

5) Prove that

$$\prod_{i=2}^n \left(1 - \frac{1}{i^2}\right) = \frac{n+1}{2n}$$

for all $n \geq 2$.

6) Suppose $f : X \rightarrow Y$ is onto and $A \subseteq Y$. Prove that

$$f(f^{-1}(A)) = A.$$