MTHT 530 Analysis for Teachers II
Problem Set 1

Due: Wednesday January 18

1) Verify the following limits **using the definition** of convergence.

a) $\lim_{n\to\infty} \frac{1}{6n^2+1} = 0$

b) $\lim_{n\to\infty} \frac{3n+1}{2n+5} = \frac{3}{2}$

c) $\lim_{n\to\infty} \frac{n+3}{n^3+4} = 0$

Hint: It might be useful to note that $n+3 \leq 2n$ for $n \geq 3$ and $n^3+4 > n^3$ for all n.

d)) $\lim_{n\to\infty} \left(\sqrt{n+1} - \sqrt{n}\right) = 0.$

Hint: It might be useful to note that

$$\left(\sqrt{n+1} - \sqrt{n}\right)\left(\sqrt{n+1} + \sqrt{n}\right) = 1.$$

2) Consider the sequence

$$0, 1, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, \ldots$$

where each string of zeros has one more zero than the previous. Does this sequence converge or diverge? If it converges to a limit L, prove that it converges to L. If it diverges, prove that it diverges.

3) Let $a_1 = 1$ and for each $n \in \mathbb{N}$ define

$$a_{n+1} = \frac{3a_n + 4}{4}.$$

a) Use induction to prove that the sequence satisfies $a_n < 4$ for all $n \in \mathbb{N}$.

b) Prove $(a_1, a_2, \ldots)$ is increasing.

c) Prove that the sequence $(a_n)_{n=1}^{\infty}$ converges.