1) Decide if the following series coverges or diverges. Justify your answers and state explicity which test or tests you are using.

a) \[ \sum_{n=1}^{\infty} \frac{n^2}{3^n} \]

b) \[ \sum_{n=2}^{\infty} \frac{1}{(\ln(n))^2} \]

c) \[ \sum_{n=2}^{\infty} \frac{1}{n \ln n} \]

d) \[ \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2} \]

e) \[ \sum_{n=1}^{\infty} \frac{n^5}{2^n + n^2} \]

2) a) Show that if \( a_n > 0 \) for all \( n \) and \( \lim na_n = L \neq 0 \), then \( \sum a_n \) diverges.

b) Assume \( a_n > 0 \) and \( \lim n^2a_n \) exists. Show that \( \sum a_n \) converges.

3) Suppose \( \sum a_n \) and \( \sum b_n \) converge. Show that

\[ \sum_{n=1}^{\infty} (a_n + b_n) = \sum_{n=1}^{\infty} a_n + \sum_{n=1}^{\infty} b_n. \]