MTHT 530 Analysis for Teachers II
Problem Set 2

Due Wednesday January 25

1) Use the definition of limits to prove the following.
   a) \( \lim_{x \to 3} 3x + 2 = 17 \)
   b) \( \lim_{x \to 2} x^3 = 8 \)

2) Let
   \[
   f(x) = \begin{cases} 
   x & \text{if } x \in \mathbb{Q} \\
   0 & \text{if } x \notin \mathbb{Q}
   \end{cases}
   \]
   Prove that \( \lim_{x \to 0} f(x) = 0 \).

3) Give an example of each of the following or prove that it is impossible.
   a) A sequence where no \( a_n \) is 0 or 1 but there are subsequences converging to both values.
   b) A monotone sequence that diverges but has a convergent subsequence.
   c) An unbounded sequence with a convergent subsequence.
   d) A sequence that has a subsequence that is bounded but containing no convergent subsequence.

4) Suppose \((a_n)_{n=1}^\infty\) is a bounded sequence that does not converge. Prove that there are convergent subsequences \((b_n)_{n=1}^\infty\) and \((c_n)_{n=1}^\infty\) that converge to different limits.

5) (5pt BONUS) Let \((a_n)_{n=1}^\infty\) be a sequence such that for all \( q \in \mathbb{Q} \) there is an \( n \) such that \( a_n = q \). Prove that for every \( r \in \mathbb{R} \), there is a subsequence converging to \( r \).