MTHT 530 Analysis for Teachers II
Problem Set 2

Due Wednesday January 25

1) Use the defintion of limits to prove the following.

a) $\lim_{x\to 5} 3x + 2 = 17$

b) $\lim_{x\to 2} x^3 = 8$

2) Let

$$f(x) = \begin{cases} x & \text{if } x \in \mathbb{Q} \\ 0 & \text{if } x \notin \mathbb{Q} \end{cases}.$$

Prove that $\lim_{x\to 0} f(x) = 0$.

3) Give an example of each of the following or prove that it is impossible.

a) A sequence where no a_n is 0 or 1 but there are subsequences converging to both values.

b) A monotone sequence that diverges but has a convergent subsequence.

c) An unbounded sequence with a convergent subsequence.

d) A sequence that has a subsequence that is bounded but containing no convergent subsequence.

4) Suppose $(a_n)_{n=1}^{\infty}$ is a bounded sequence that does not coverge. Prove that there are convergent subsequences $(b_n)_{n=1}^{\infty}$ and $(c_n)_{n=1}^{\infty}$ that converge to different limits.

5) (5pt BONUS) Let $(a_n)_{n=1}^{\infty}$ be a sequence such that for all $q \in \mathbb{Q}$ there is an n such that $a_n = q$. Prove that for every $r \in \mathbb{R}$, there is a subsequence converging to r.