MTHT 530 Analysis for Teachers II
Problem Set 2

Due Wednesday February 1

Do Problems 3 and 5 from Chapter 7 of Spivak's *Calculus*. The solutions are in the back so you need only turn in 3 ii).

1) Let $f, g : [-1, 1] \to \mathbb{R}$. Suppose f is is bounded (i.e., there is M such that $|f(x)| \leq M$ for all $x \in [-1, 1]$), g is continuous at 0 and $g(0) = 0$. Let $h(x) = g(x)f(x)$.

Prove that h is continuous at 0.

2) Let

$$f(x) = \begin{cases} 0 & \text{if } x \notin \mathbb{Q} \\ \frac{1}{n} & \text{if } n \in \mathbb{N} \text{ is least such that there is } m \in \mathbb{Z} \text{ with } x = \frac{m}{n} \end{cases} .$$

For example, $f(0) = 1$, $f(n) = 1$ for each integer n,

$$f(1/4) = f(3/4) = f(-7/4) = 1/4 \ldots$$

a) Prove that $\lim_{x \to a} f(x) = 0$ for all $a \in \mathbb{R}$.

b) For which a is f continuous at a?

3) Suppose $f : [0, 1] \to \mathbb{R}$ and $g : [0, 1] \to \mathbb{R}$ are continuous and $f(x) > g(x)$ for all $x \in [0, 1]$. Prove that there is $a > 0$ such that $f(x) \geq g(x) + a$ for all $x \in [0, 1]$.

4) Suppose $f : [a, b] \to \mathbb{R}$ is continuous. Prove that there are c, d such that f maps $[a, b]$ onto $[c, d]$.