

Midwest Model Theory
University of Illinois at Chicago
Abstracts

Friday April 6 UIC Mathematics Colloquium

SPEAKER: Simon Thomas (Rutgers)

TITLE: Superrigidity and the classification problem for the torsion-free abelian groups of finite rank

ABSTRACT: In 1937, Baer solved the classification problem for the torsion-free abelian groups of rank 1. Since then, despite the efforts of such mathematicians as Kurosh and Malcev, no satisfactory solution of the classification problem has been found for the torsion-free abelian groups of rank $n \geq 2$. So it is natural to ask whether the classification problem is genuinely more difficult for the groups of rank $n \geq 2$. In this talk, I will explain how this question can be answered, using Zimmer's superrigidity theorem.

Saturday April 7

10:00-11:00 Chris Laskowski (Maryland), *Elementary diagrams of trivial, strongly minimal models are model complete*

ABSTRACT: I will sketch the proof of this result and explain how it arose from questions in recursive model theory.

11:10-12:10 Anand Pillay (Urbana), *p-jets of varieties over differential and difference fields*

ABSTRACT: (Joint work with M. Ziegler.) I explain how the consideration of linear differential (difference) equations on p-jets of varieties over differential (difference) fields yields all the structural results (and more) relevant to the diophantine-geometric applications.

1:30-2:30 Denis Hirschfeld (U. Chicago), *Computable Presentations and Structure*

ABSTRACT: Computable model theory includes the study of the extent to which theorems of model theory hold in effectivized settings, but it also includes a body of results that constitutes an independent theory based on effectivized versions of fundamental concepts of model theory, in particular that of structure. One of the central concerns of this computable structure theory is the analysis of the different ways in which a structure may be effectivized. In this talk, I will survey a range of results connected with this topic, with the aim of introducing some of the questions and methods of computable structure theory. I will also place many of these results in the framework of a contrast between cases in which we can prove structure theorems that help us understand the relationships between the various computable versions of a structure and cases in which we can show that no such theorems are possible. I will discuss one way to achieve such nonstructure results by using an effective kind of reduction between structures related to the idea of interpretation.

2:40-3:40 Simon Thomas (Rutgers), *The classification problem for p -local torsion-free abelian groups of finite rank*

ABSTRACT: Let $n \geq 3$. In this talk, I will explain why the classification problems for the p -local torsion-free abelian groups of rank n are incomparable with respect to Borel reducibility for distinct primes p . (Recall that an abelian group A is said to be p -local iff A is q -divisible for every prime $q \neq p$.)

4:00-5:00 Jessica Young (MIT). *On Martin's Conjecture*

ABSTRACT: For a first order theory T let L^* be the smallest language containing the non-principal types of T . Martin conjectured that any theory with uncountably many countable models will have continuum many extensions in L^* . This is stronger than Vaught's conjecture. We provide a counterexample to Martin's conjecture.

Sunday April 7

9:30-10:30 Hans Schoutens (Ohio State), *Lefschetz categories, the Ax-Kochen-Ershov Principle and transfer from prime to mixed characteristic*

ABSTRACT: For more than two decades, homological algebra in equicharacteristic is now well understood, due to work of Peskine-Szpiro on the action of Frobenius, work of Hochster on big Cohen-Macaulay algebras, and more recently, work of Hochster-Huneke in tight closure theory. In contrast, much less progress has been made for the mixed characteristic analogs (one break-through was Robert's New Intersection Theorem). As a consequence, many homological conjectures are still open. I will sketch a program that would try to solve these conjectures at least asymptotically, that is to say, for large enough characteristic of the residue field (depending only on the degrees of the polynomials involved in the representation of the data and not on the data itself).

The approach is non-standard and is inspired by my previous work on tight closure theory of the non-standard Frobenius over the complex numbers. The starting point now is the Theorem of Ax-Kochen-Ershov that the ultraproduct Z^* of the p -adic integers is the same as the ultraproduct of the power series rings in a single variable over the p element fields. If instead we take ultraproducts of polynomial rings over these discrete valuation rings, then we get two non-comparable objects A_{eq}^* and A_{mix}^* . Fortunately, these two rings have a common subring A , the polynomial ring over Z^* . The key idea is now the following transfer principle. The homomorphic images of A_{eq}^* form an example of a Lefschetz category, that is to say, a category consisting of ultraproducts of rings of prime characteristic. In particular, on each such ring, we have the action of the ultraproduct of the Frobenii and this allows for a 'tight closure like' treatment of various homological conjectures. We then descend to the subring A ; this is not always easy since $A \rightarrow A_{eq}^*$ is not faithfully flat (as it was when the base rings are fields instead of discrete valuation rings). Next, we extend the results in the overring A_{mix}^* (for which we are facing a similar problem due to the lack of flatness). Finally, we use Los's Theorem to descend the result to the mixed characteristic case.

I will explain the process by proving an asymptotic version of the Hochster-Roberts Theorem over the p -adic integers.

10:40-11:40 Yevgeniy Vasilyev (Urbana), *Generic pairs and geometry of SU-rank 1 structures*

ABSTRACT: The generic pair construction, which is shown to work well in the general simple case in a joint work with Itay Ben-Yaacov and Anand Pillay, was originally introduced in the supersimple SU-rank 1 context, where generic pairs have certain geometric applications, similarly to the pairs of strongly minimal structures used by Steven Buechler in his proof that pseudolinearity implies local modularity. We will discuss the applications concerning linearity (1-basedness) and pseudolinearity in the SU-rank 1 case, in particular, the relation between these properties, combinatorial geometry and the SU-rank of the generic pairs.

11:50-12:50 Ben-Yaacov (Paris/Urbana), *Generic pairs of simple cats produce simple kittens*

ABSTRACT: [Joint work with Anand Pillay & Evgueni Vassiliev]

Generic pairs are to simple theories what beautiful pairs are for stable ones. In the stable case there is a dichotomy: either T does not have the f.c.p., and the theory of beautiful pairs is stable and otherwise well behaved; or it does, and the theory of beautiful pairs is "wild". A similar dichotomy exists in the simple case. We adopt an alternative approach, proving that for every simple T the theory T^P of its generic pairs is well behaved, but is not necessarily of first order. Instead, T^P is a compact abstract theory (cat). In this construction no pathologies arise, and in particular if T is simple or stable so is T^P .