

Stat/Econ 473: Game Theory
Problem Set 3

Due: Friday September 19

Do the following problems from the text: 5.5, 5.6

For a 5pt bonus do problem 5.3 b).

Clarification In problem 5.5 your contribution should be taken away from the value of the final result. For example, if you contribute and the streetlight is built your payoff will be $2=3-1$ and if it is not built your payoff will be -1 .

1) Two people enter a bus. Two adjacent cramped seats are free. Each person must decide whether to sit or stand. Sitting alone is more comfortable than sitting together which is more comfortable than standing.

a) Suppose each person cares only about her own comfort. Model this situation in a strategic game and find all pure strategy Nash equilibria.

b) Suppose each person is altruistic and ranks the outcomes based on the other person's comfort, but prefers to stand if the other person stands.

2) Watch the following excerpt from the film *A Beautiful Mind*, the Academy Award winning film about the life of John Nash.

<https://www.youtube.com/watch?v=CemLiSI5ox8>

Consider the (admittedly sexist) strategic game where 4 men meet 5 women, one blonde and four brunettes, in a bar. Each man must decide to make a pass at the blonde or any of the brunettes. Each man prefers the blonde to the brunettes and prefers the brunettes to failure. If more than one man makes a pass at the blonde everyone will fail. If at most one man makes a pass at the blonde everyone will succeed.

a) Is the profile where every man makes a pass at a brunette a Nash equilibrium?

b) What are the pure strategy Nash equilibria? Did the screenwriters get this right?

3) (Cournot oligopoly) Consider an economy where there are N -firms. Each firm chooses to produce q_i . Let $Q = q_1 + \dots + q_n$ be the total production and let $p(Q) = \alpha - Q$ be the price each unit will sell for if Q items are produced. Assume that the firms are identical and each has a cost function $C(q_i) = cq_i$. Determine the best response functions for each firm and find a Nash equilibrium (q_1^*, \dots, q_N^*) where $q_1^* = \dots = q_N^*$. What happens to the equilibrium price as $N \rightarrow +\infty$? For those of you in Economics why is this what you'd expect? [For 3 bonus points show that the only equilibrium is one where $q_1^* = \dots = q_N^*$].