

Stat/Econ 473 Game Theory
 Problem Set 10

Due: Thursday April 14

1) The Sheriff (Player 1) approaches a suspicious character (Player 2). Each has to decide simultaneously whether to draw their gun (D) or wait (W). There is a probability p that the suspicious character is a sharpshooter and probability $1 - p$ that he is an amateur. The suspicious character knows his skill level, but the Sheriff does not.

If Player 2 is a sharpshooter the payoffs are

	D	W
D	2,3	3,1
W	1,4	8,2

If Player 2 is an amateur the payoffs are

	D	W
D	5,2	4,1
W	6,3	8,4

Find all pure strategy Bayes–Nash equilibria.

2) Consider the following version of Battle of the Sexes. There is $1/2$ chance that Alice wants to meet Bob and a $1/2$ change she wants to avoid Bob. If Alice wants to meet Bob, the payoffs are

	F	O
F	1,2	0,0
O	0,0	2,1

If Alice wants to avoid Bob, the payoffs are

	F	O
F	0,2	1,0
O	2,0	0,1

Suppose there is a $1/2$ chance that Bob knows whether or not Alice wants to meet him or avoid him (and the probability that Alice wants to meet Bob is independent of whether or not Bob knows what Alice wants). [Hint: Model this as a game where there are two types of Alices (Alice1 wants to meet Bob and Alice2 wants to avoid him) and three types of Bobs (Bob1 doesn't know what Alice wants, Bob2 knows Alice wants to meet him, Bob3 knows Alice wants to avoid him).]

a) Calculate the posterior probabilities for each type of Player. (i.e., What is the probability Alice is of type i given Bob is of type j ? What is the probability Bob is of type j given that Alice is of type i).

- b) Find the best responses for each type of Alice to all possible strategies for Bob.
- c) Find the best responses for each type of Bob to all possible strategies for Alice.
- d) Find all pure strategy Bayes–Nash equilibria.

3) Two animals compete for a location. Each animal can be either strong or weak. There is a $1/2$ probability that both are strong, a $1/3$ probability that both are weak and a $1/6$ probability that the first is weak and the second is strong.

Each animal must decide to be aggressive (A) or passive (P).

- If both are passive the payoffs are 0 for each animal;
- if one is aggressive and the other is passive, the aggressive animal gets 6 and the passive gets -2.
- if both are aggressive the payoffs depend on their strengths, if both are strong they each get -3 if both are weak they each get -1, while if one is strong and the other is weak the strong gets 3 and the weak gets -6.

Model this a Bayesian game where there are two types of Animal 1 (1S, 1W) and two types of Animal 2 (2S,2W).

- a) Write down the payoff matrices for each combination of types of Animal 1 and Animal 2
- b) Calculate the posterior probabilities for each type of Player.
- c) Find the best responses for each type of Animal 1.
- d) Find the best responses for each type of Animal 2.
- e) Find all pure strategy Bayes–Nash equilibria.