Stat/Econ 473 Game Theory
Problem Set 12

Due: Thursday April 28

Problem Set 12 is optional. If you submit problem set 12 your grade can be used to replace your third lowest homework score–recall that the lowest two homeworks are dropped.

1) In a second price sealed bid auction with reserve price r. If all bids are below r, then object is not sold. If at least one bid is above r the highest bidder gets the object and pays the larger of r and the second highest bid.

Suppose there are two potential buyers, each with valuation that is randomly chosed from 0,1,2 each occuring with equal probaility 1/3 and assume that buyer's valuations are independent. Assume that if both players make the same bid $x \geq r$ then they each get the item at price x with probability 1/2.

a) Is it still a dominant strategy for each player to bid their valuation if $r > 0$? Justify your answer.

b) What is the expected revenue to the seller if $r = 0$?

c) What is the expected revenue to the seller if $r = 1$?

d) What is the expected revenue to the seller if $r = 2$?

e) What do you conclude about the value to the seller of using a reserve in this auction?

2) Suppose there are two bidders each of whom value an object at either 4 (low) or 8 (high) with probability 1/2 each, and the valuations of the players are independent. Suppose they bid for the object in a first price sealed bid auction and if they make the same bid then the each win the auction with probability 1/2. There bids may be any real number between 0 and 8.

a) Show that in a Bayes–Nash equilibrium a player who values the object at 4 will never use a pure strategy "bid b" where $b > 4$.

b) Show that in a Bayes–Nash equilibrium a player who values the object at 4 will never use a pure strategy "bid b" where $b < 4$.

c) Show that a player who values the object at 4 would never use a mixed strategy where we bid < 4 with positive probability. [Hint: If Player 1 used such a strategy then Player 2 could get an advantage be bidding slightly hire than the expectation.] Conclude that a Player who values the object at 4 must bid 4 in equilibrium.

d) Show that in equilibrium a player with high value should never bid $b > 6$. [Hint: Use the observation from c)].

e) Show that there is no pure strategy Bayes–Nash equilibrium.

f) (Bonus) i) Show that there is a Bayes–Nash equilibrium where a player who values the object at 4 bids 4 and a player who values the object at 6 chooses a bid b where $4 \leq b \leq 6$ is chosen randomly using the distribution where the probability that $b \leq r$ is $\frac{b-4}{8-b}$. [In fact this is the only possible Bayes–Nash equilibrium]

ii) What is the expected revenue for the seller in this equilibrium? How does this compare to the expected revenue in a second price sealed bid auction? (Note: *The Revenue Equivalence Theorem does not apply here because there is not a continuous distribution of values.*)