**Stat/Econ 473 Game Theory**
Problem Set 8

**Due: Tuesday March 29**

**From the Text:** Do problems: 15.15-15.16

1) Consider the following version of Prisoner's Dilemma repeated infinitely many times with discount factor $\delta$.

| | C | Q |
|---|---|---|
| C | 0,0 | 7,-2 |
| Q | -2,7 | 5,5 |

Is there a subgame perfect equilibrium where Player 1 confesses at each stage and Player 2 stays quiet at each stage. If yes, describe the strategy and determine how large $\delta$ needs to be.. If no, why not?

2) Consider the following game from Problem Set 7 repeatedly infinitely often with discount factor $\delta$.

| | L | C | R |
|---|---|---|---|
| T | 10,10 | 2,12 | 0,13 |
| M | 12,2 | 5,5 | 0,0 |
| B | 13,0 | 0,0 | 1,1 |

Is there a subgame perfect equilibrium where we play $(T, L)$ in each round. If yes, describe the strategy and determine how large $\delta$ needs to be.. If no, why not?

3) Consider the following version of the treasury bill auction from Chapter 14 (that we also discussed in class) Each week the government auctions off 100 units of treasure bills. Assume there are two firms bidding on the treasury bills. Each wants 75 units. They either bid at a high price H or a low price. This is a second price sealed bid auction. If both bid high, the each get 50 units at a high price. If both bid low, they each get 50 units at the low price. If one bids high and the other bids low, the high bidder get 75 units at the low price and the low bidder gets 25 units at the low price. Assume that the firm's profit at the low price is 15 per unit and the profit at the high price is 10 per unit.

a) Write down the strategic form of each individual auction and find the Nash equilibrium.

b) Suppose the auction is repeated one a week for a year. What is the subgame perfect equilibrium?

c) Suppose that each time the auction is held there is a probability $\delta$ where $0 < \delta < 1$ that there will be another auction and the auctions may continue indefinitely. Find strategies for each firm such that, for $\delta$ sufficiently close to

1, there is a subgame perfect equilibrium where each firm bids low in every auction.

d) For what values of $\delta$ is your solution to c) a subgame perfect equilibrium.

4) Consider an industry where there are two firms. Firm $i$ decides to produce $q_i$. Firm 1 chooses $q_1$ first and then Firm 2, knowing $q_1$ chooses $q_2$. The inverse demand function is $p = 10 - q_1 - q_2$. The cost of production is 1 per unit for each firm.

a) Find the best response functions for Firm 1 and Firm 2.

b) Find the Stackelberg equilibrium $(q_1^*, q_2^*)$. What are the profits for each firm?

c) Suppose Firms 1 and 2 collude and decide to produce $\widehat{q}_1 = 4.2$ and $\widehat{q}_2 = 2.3$ What are the profits in this case?

d) What is the best response for player 2 to $\widehat{q}_1$? What are the profits for each firm when Firm 1 uses $\widehat{q}_1$ and Firm 2 makes a best response?

e) Consider the extensive game where Firm 1 moves first and it cooperates (C) it sets its price at $\widehat{q}_1$ and if it maximizes (M) it sets its price at $q_1^*$. If Firm 1 cooperates, then Firm 2 chooses to either cooperate (C) and set the price at $\widehat{q}_2$ or maximize (M) and set the price at the best resposnes to $\widehat{q}_1$. If Firm 1 decides to maximize (M), then Firm 2 will play $q_2^*$ which is a best response to $q_1^*$.

Draw the game tree and find the subgame perfect equilibrium in this game.

f) Suppose the game from part e) is repeated 10 times. What are the possible subgame perfect equilibria?

g) Suppose the game from part e) is repeated indefinitely where each time you play there is probability $0 < \delta < 1$ that you will play again. Show that for $\delta$ sufficiently close to 1 there is a subgame perfect equilibrium where both firms choose the cartel quantity in every round. Carefully describe the strategies for each Player. How large does $\delta$ have to be for this to be a subgame perfect equilibrium?

5) Consider the following game $G$ repeatedly infinitely often with discount factor $\delta$.

| | L | C | R |
|---|---|---|---|
| T | 0,2 | 0.5,2 | 3,1 |
| M | -2,4 | 0,3 | 4,3 |
| B | -1,3.5 | 1,1 | 4,0 |

a) Find all pure strategy Nash equilibria in $G$.

b) Graph the feasible set for $G$.

c) Graph and describe the set of $(x, y)$ such that for all $\epsilon > 0$ and $\delta$ sufficiently close to 1 there is a subgame perfect equilibrium with average payoffs within $\epsilon$ of $(x, y)$.