Cartels in Repeated Games

We consider an industry where there are two identical firms. Firm i chooses to produce q_i. The total production is $Q = q_1 + q_2$. We assume the inverse demand function is $p = 100 - Q$ and that each firm has cost function $c(q) = q$.

The profit function for each firm is

$$\pi_i(q_1, q_2) = (100 - q_1 - q_2)q_i - q_i.$$

We first look for what happens in Cournot–Nash equilibrium. To find the best response function for Firm i we solve

$$\frac{\partial \pi_i}{\partial q_i} = 0.$$

For Firm 1,

$$\frac{\partial \pi_1}{q_1} = 100 - 2q_1 - q_2 = 0.$$

This gives a best response function

$$BR_1(q_2) = \frac{99 - q_2}{2}.$$

Similarly for Firm 2, we get

$$\frac{\partial \pi_2}{q_2} = 100 - q_1 - 2q_2 = 0.$$

This gives a best response function

$$BR_2(q_1) = \frac{99 - q_1}{2}.$$

Solving the equations

$$2q_1 + q_2 = 99$$

$$q_1 + 2q_2 = 99$$

we find that $q_1 = q_2 = 33$.

In this case both firms have profit $\pi_1 = \pi_2 = (100 - 66) * 33 - 33 = 1089$.

We contrast that the case where the two firms collude and form a cartel. They choose Q to maximize total profits and then decide to produce $q_1 = q_2 = Q/2$. The monopoly profit function is

$$\pi(Q) = (100 - Q)Q - Q$$

which we maximize by solving $\frac{d\pi}{dQ} = 0$. Thus

$$\frac{d\pi}{dQ} = 99 - 2Q = 0$$

So $Q = 49.5$ and $q_1 = q_2 = 24.75$. In this case $\pi(Q) = 1225.125$.

As we saw before, in the cartel solution neither player making a best response.

$$BR_1(24.75) = 37.125$$

and

$$\pi_1(37.125, 24.75) = (100 - 37.125 - 24.75) * 37.125 - 37.125 \approx 1378.266.$$

Thus, if Firm 2 is using the cartel price, Firm 1 would do much better by making its best response

We now come to our main question. *Is it possible that if we play this game repeatedly we can reach an equilibrium where the cartel solution is used in every round?* Suppose we repeat these decisions infinitely often with discount factor $0 \leq \delta < 1$.

Consider the grim trigger strategy profile where each player uses the following strategy: Begin playing the cartel solution. If both firms have used the cartel solution (produce 24.75) in every previous round, then continue to use the cartel solution. If not, use the Cournot–Nash equilibrium (produce 33) in every round.

If both firms follow this strategy, each firm will use the cartel solution in each round. The payoff will be 1225.125 in each round and the discounted profit is

$$\sum_{n=0}^{\infty} 1225.125\ \delta^n = \frac{1225.125}{1-\delta}.$$

Can either firm gain by deviating from this strategy? If they do, it would be optimal to deviate in the first round. In this case, they would produce 37.125 in the first round and get a profit in that round of 1378.266. In all later rounds the other firm will produce 33 and their best response is also to produce 33 so that in all later rounds they would receive a profit of 1089. The discounted profit would is

$$1378.266 + \sum_{n=1}^{\infty} 1089\ \delta^n = 1378.266 + \frac{1089\ \delta}{1-\delta}.$$

The grim trigger strategy produces an equilibrium if

$$\frac{1225.125}{1-\delta} \geq 1378.266 + \frac{1089\ \delta}{1-\delta}$$

$$1225.125 \geq 1378.266(1-\delta) + 1089\delta$$

$$289.266\delta \geq 153.141$$

$$\delta \geq 0.529$$

Thus is $\delta \geq 0.529$, the grim trigger strategies give a subgame perfect equilibrium where each firm chooses to produce the cartel amount in each round.