Pareto Optimality

Suppose we are considering a game with N players. A strategy profile $(a_1, \dots, a_N)$ with payoff vector $(x_1, \dots, x_N)$ is *Pareto optimal* if there is no other strategy profile $(b_1, \dots, b_N)$ with payoff vector $(y_1, \dots, y_N)$ where:
i) $x_i \le y_i$ for all $i = 1, \dots, N$ (i.e., no player does worse)
ii) $x_j < y_j$ for some j (some player does better).

While Pareto optimal outcome are desirable and might be expected in situations where the players cooperate to reach a final outcome, they may or may not agree with Nash equilibria.

Examples
1) Prisoner's Dilema

	C	Q
C	-3,-3	0,-10
Q	-10,0	-1,-1

In this case the Pareto optimal profiles are (C,Q), (Q,C) and (Q,Q). While the Nash equilibrium is (C,C). This shows that even a dominant solution need not be Pareto optimal.

2)

	L	R
T	2,2	0,0
B	0,0	1,1

In this case (T,L) and (B,R) are Nash equilibria and (T,L) is the unique Pareto optimal profile.

3)

	L	R
T	2,2	1,1
B	1,1	0,0

In this case (T,L) is a dominant solution, and hence a Nash equilibrium. It is also the unique Pareto optimal profile.

4)

	L	C	R
T	3,3	1,4	0,0
M	4,1	2,2	0,0
B	0,0	0,0	1,1

In this case (T,L), (M,L), (T,C) and (B,C) are Pareto optimal, while (B,B) is the unique pure strategy equilibrium.