Stat/Econ 473 Game Theory
Very Simple Poker Model

Consider the following very simplified version of poker. Suppose we have a deck with 3 cards A, K, Q, where and A beats a K or Q and a K beats a Q. Each player is dealt one card from the deck.

Player 1 decides to check (C) or bet (B)

If Player 1 decides to check, the high card gets 1 and the low card gets -1.

If Player 1 decides to bet, Player 2 decides to call (c) or fold (f). If 1 bets and 2 calls, the high card gets 2 and the low card gets -2. If Player 2 folds, Player 1 gets 1 and Player 2 gets -1.

There are three types of Player 1: 1A, 1K, 1Q depending on what card they are dealt and there are three types of Player 2: 2A, 2K 2Q.

The following is the prior distribution of probabilities:

	2A	2K	2Q
1A	0	1/6	1/6
1K	1/6	0	1/6
1Q	1/6	1/6	0

The posterior probabilities are easy to calculate. For example:

$$Pr(2A|1A) = 0, Pr(2K|1A) = 1/2 \text{ and } Pr(2Q|1A) = 1/2.$$

The other calculations are similar knowing what card Player 1 has there is a 1/2 chance 2 has each of the other cards and 0 chance 2 holds the same card.

Each type of Player 1 has 2 choices C or B. Each type of Player 2 has two choices call or fold when Player 1 bets. This give 8 strategies for each Player, but we start by simplifying by IDWDS.

• For Player 1A: B weakly dominates C–If Player 1A checks she always gets 1, if she bets she will always get at least 1 and might get 2.

• For Player 2A: c weakly dominates f–if player 2A calls, he will get 2, if he folds he will get -1.

• For Player 2Q: f weakly dominates c–if player 2Q calls, he will get -2, while if he folds he will get 1.

• Since Player 2A will call and Player 2Q will fold, Player 1K has expectation 0 if he checks and expectation 1/2(-2)+(1/2)(1) if he bets. Thus Player 1K should always check.

This leaves the question of what should 1Q and 2K do?

If Player 1 is using the strategy B/C/C (i.e., only betting with an A), then Player 2K should fold. Suppose Player 2 is using the strategy c/f/f (ie. only calling with an A). Then

$$u_{1Q}(B, c/f/f) = 1/2(-2) + 1/2(1) = -1/2$$

while

$$u_{1Q}(C, c/f/f) = -1.$$

Thus Player 1Q would prefer to B with a Q. So there is no pure strategy equilibrium where Player 1 uses the strategy B/C/C.

If Player 1 is using the strategy B/C/B (always bluffing with a Q), then

$$u_{2K}(B/C/B, c) = 1/2(-2) + 1/2(2) = 0 \text{ while } u_{2K}(B/C/B, f) = -1.$$

Thus Player 2 will use c/c/f. Against, c/c/f

$$u_{1Q}(B, c/c/f) = 1/2(-2) + 1/2(-2) = -2 \text{ and } u_{1Q}(C, c/c/f) = -1.$$

Thus 1Q would prefer to C. So there is no pure strategy equilibrium where Player 1 uses B/C/B.

We look for a mixed strategy equilibrium. Here is the strategic form of the game.

	c/c/f	c/f/f
B/C/B	-1/6, 1/6	1/6, -1/6
B/C/C	1/6, -1/6	0, 0

How do we compute the expected payoffs? Note that the payoff to Player 2 is alway - the payoff to Player 1 so we just compute the payoffs for Player 1.

$$u_1(B/C/B, c/c/f) = 1/6(2 + 1 - 1 + 1 - 2 - 2) = -1/6$$
$$u_1(B/C/B, c/f/f) = 1/6(1 + 1 - 1 + 1 - 2 + 1) = 1/6$$
$$u_1(B/C/C, c/c/f) = 1/6(2 + 1 - 1 + 1 - 1 - 1) = 1/6$$
$$u_1(B/C/C, c/f/f) = 1/6(1 + 1 - 1 + 1 - 1 - 1) = 0$$

Suppose Player 2 uses the mixed strategy σ_q: Play c/c/f with probability q and c/f/f with probability $1 - q$. Then

$$u_1(B/C/B, \sigma_q) = q(-1/6) + (1 - q)(1/6) = \frac{1 - 2q}{6}$$

and

$$u_1(B/C/C, \sigma_q) = q(1/6).$$

Player 1 is indifferent if $q = 1/3$.

Suppose Player 1 uses a mixed strategy σ_p: Play B/C/B with probability p and $B/C/C$ with probability $(1 - p)$. Then

$$u_2(\sigma_p, c/c/f) = p(-1/6) + (1 - p)(1/6) = \frac{1 - 2p}{6}$$

and

$$u_1(\sigma_p, c/f/f) = p/6.$$

Player 2 is indifferent if $p = 1/3$.

Thus there is a Bayes-Nash equilibrium when $p = q = 1/3$.

What does this mean? Optimally, Player 1 should bluff and bet with a Q 1/3 of the time and Player 2 should call with a K 1/3 of the time. Note that in this case Player 1 has an expected payoff of 1/18 and Player 2 has an expected payoff of -1/18.