Homework 4

• Let $T(x)$ be the Thomae function. Find a continuous function $h : [0,1] \rightarrow \mathbb{R}$ such that $h(x) \geq T(x)$ for all $x$ and $\int_0^1 h < 1/2$.

• Let $F(x)$ be the function

$$F(x) = \int_0^x \frac{1}{1+t^2}dt + \int_0^{\frac{1}{2}} \frac{1}{1+t^2}.$$  

Show that $F(x)$ is a constant.

• Show that $\int_0^\infty \frac{1}{1+x^2}$ exists and calculate it using the face that $(\arctan(x))' = \frac{1}{1+x^2}$.