Sample Exam

• Let $f(x) = x^2$ on $[0, 1]$. Let $P$ be the partition $P = \{0, 1/4, 1/2, 3/4, 1\}$. Find $U(f, P)$ and $L(f, P)$.

• Give the definition of what it means for $f$ to be integrable on $[a, b]$. From the definition show that the function $f(x)$ defined on $[0, 1]$ to be 0 if $x \neq 1/2$ and 1 if $x = 1/2$ is integrable.

• a) Suppose $f$ is continuous on $[0, 1]$ with $\int_0^1 f(x) \, dx = 1/2$. Show that there exists $0 \leq c \leq 1$ such that $f(c) = 1/2$. b) Give an example where the above is false if $f$ is not continuous at some point in $[0, 1]$.

• Does there exist an integrable function $f(x)$ such that $t(x) = \int_0^x f(t) \, dt$ where $t(x)$ is the Thomae function?