HOMEWORK
MATH 320

8/25/14

(1) Let $f\colon \mathbb{R} \to \mathbb{R}$ be a linear function, that is for all $x, y, a \in \mathbb{R}$, $f(x+y) = f(x) + f(y)$ and $f(ax) = af(x)$. Prove that there exists $m \in \mathbb{R}$ such that for all $x \in \mathbb{R}$,

$$f(x) = mx$$

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(1) Section 1.2: # 2, 3, 6.
(2) Solve the system of equations:

$$\begin{aligned} 3x + y &= 1 \\ 2x + 3y &= -1 \end{aligned}$$

(3) Solve the system of equations:

$$\begin{aligned} x_1 + x_2 - 3x_3 &= 3 \\ -2x_1 - x_2 &= 4 \\ 4x_1 + 2x_2 + 3x_3 &= 7 \end{aligned}$$

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(1) Section 1.3: # 2, 3, 5, 6.

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(1) Section 1.4: # 1, 3.
(2) Row-reduce the following matricies:
   (a) $\begin{pmatrix} 3 & -1 & 4 \\ 2 & -\frac{1}{2} & 1 \end{pmatrix}$
   (b) $\begin{pmatrix} 1 & 1 & -1 \\ 3 & -1 & 1 \\ 1 & -3 & 3 \end{pmatrix}$
   (c) $\begin{pmatrix} 3 & -1 & 2 \\ -4 & 1 & -2 \\ 2 & -2 & 4 \\ 0 & 3 & -6 \end{pmatrix}$

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(1) Section 1.4: # 4, 5, 6, 7, 8, 10

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(1) Section 1.5: # 1, 2, 3, 7.

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(1) Section 1.5: # 4, 5. Section 1.6 # 1.

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(1) Section 1.6 # 3, 4, 5, 6, 7, 8.

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(1) Suppose the system $AX = Y$ has a solution for any Y. Prove that the system $AX = 0$ has *only* the trivial solution.

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(1) Section 2.1 # 1, 2, 4, 5, 6, 7.

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(1) Section 2.2 # 1, 2, 5.

(2) Let V be a vector space over a field F, and let W be a non-empty subset of V such that for all $\alpha, \beta \in W$ and all $c \in F$, $c\alpha + \beta \in W$. Prove that W is a subspace of V.

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(1) Section 2.2 # 3, 4, 6.

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(1) Section 2.2 # 1, 2, 3, 4, 5, 6.

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(1) Section 2.3 # 1, 2, 3, 4, 5, 6, 9, 10, 12.

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(1) Section 2.6 # 2, 3, 4.

(2) Let V be a vector space over F with $dim(V) = n$ Let $S \subseteq V$ such that S spans V and $|S| = n$. Prove that S is linearly independent.

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(1) Section 2.4 # 1, 2, 3, 7

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(1) Section 2.6 # 1, 5, 6, 7

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(1) Section 3.1 # 1, 9, 10.

(2) Give an example of a linear transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$.

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(1) Section 3.1 # 4, 5, 6, 11.

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(1) Section 3.1 # 2, 3, 7, 8, 12.
(2) Let $T\colon V \to W$ be a linear transformation. Prove that $Range(T)$ is a subspace of W and $Null(T)$ is a subspace of V.

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(1) Section 3.4 # 1, 2, 3, 5.

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(1) Section 3.4 # 6, 7, 12.
(2) Let $T\colon V \to W$ and $S\colon W \to U$ be linear transformations.
    (a) Prove that $S \circ T\colon V \to U$ is a linear transformation.
    (b) Suppose T is a bijection and $T^{-1}\colon W \to V$ is the *inverse function* of T, that is $(T^{-1} \circ T)(\alpha) = \alpha$ for all $\alpha \in V$ and $(T \circ T^{-1})(\beta) = \beta$ for all $\beta \in W$. Prove T^{-1} is a linear transformation.

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(1) Section 3.2 # 1, 2, 3.
(2) Let $T\colon V \to W$ be a linear transformation. Prove that T is injective if and only if $Null(T) = \{0\}$.

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(1) Section 3.2 # 6, 7, 8.
(2) Complete the sheet of practice problems posted on the previous page.

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(1) Section 3.3 # 1, 2, 4, 6.

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(1) Section 5.2 # 3, 4, 5. (Note: when the book says " Let K be a commutative ring", you can assume that K is a field).

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(1) Section 6.2 # 1, 2, 3, 4, 5.