MATH 320 **Quiz 1** **Name:**__________________________

1. Let $f\colon \mathbb{R} \to \mathbb{R}$ be a linear function, that is for all $x, y, a \in \mathbb{R}$, $f(x+y) = f(x) + f(y)$ and $f(ax) = af(x)$. Prove that there exists $m \in \mathbb{R}$ such that for all $x \in \mathbb{R}$,
$$f(x) = mx$$

2. Solve the system of equations:
$$\begin{aligned} 3x + y &= 1 \\ 2x + 3y &= -1 \end{aligned}$$

MATH 320 **Quiz 2** **Name:**__________________________

1. Prove that the following two systems of equations are *not* equivalent:

System 1:

$$\begin{aligned} 3x + y &= 0 \\ 2x + 3y &= 0 \end{aligned}$$

System 2:

$$\begin{aligned} 5x + 4y &= 0 \\ x + \frac{4}{5}y &= 0 \end{aligned}$$

MATH 320 **Quiz 3** **Name:**____________________________

1. Let
$$A = \begin{pmatrix} 6 & -4 & 0 \\ 4 & -2 & 0 \\ -1 & 0 & 3 \end{pmatrix}$$
Find all solutions of $AX = 3X$.

MATH 320 **Quiz 4** **Name:**____________________________

1. Row-reduce the coefficient matrix of the following system, then find all solutions to the system.

$$\begin{aligned} x_1 + x_2 + x_3 &= 0 \\ 3x_1 - x_2 + x_3 &= 0 \\ x_1 - 3x_2 + 3x_1 &= 0 \end{aligned}$$

MATH 320 **Quiz 5** **Name:**__________________________

1. Consider the system of equations:

$$\begin{aligned} x_1 - x_2 + 2x_3 &= 1 \\ 2x_1 \qquad + 2x_3 &= 1 \\ x_1 - 3x_2 + 4x_3 &= 2 \end{aligned}$$

   Does this system have a solution? If so, describe explicitly all solutions.

MATH 320 **Quiz 6** **Name:**__________________________

1. Give an example of a system of two linear equations in two unknowns which has no solution.

2. Give an example of a 2×2 matrix $A \neq 0$ for which $A^2 = 0$.

MATH 320 **Quiz 7** **Name:**__________________________

1. Let $A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{pmatrix}$. Find elementary matrices $E_1, ..., E_k$ such that

$$E_k ... E_1 A = I$$

MATH 320 **Quiz 8** **Name:**__________________________

1. Let
$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$
Prove, using elementary row operations, that A is invertible if and only if $(ad - bc) \neq 0$.

MATH 320 **Quiz 9** **Name:**__________________________

1. Let A be a non-invertible $n \times n$ matrix. Prove that there exists an $n \times n$ matrix $B \neq 0$ such that $BA = 0$.

MATH 320 **Quiz 10** **Name:**__________________________

1. True/False

   (a) $(AB)^{-1} = B^{-1}A^{-1}$.

   (b) If $AB = 0$ and $B \neq 0$, then A is not an invertible matrix.

   (c) $\begin{pmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is an elementary matrix.

   (d) $AX = 0$ has non-trivial solutions if and only if A is row-equivalent to the identity matrix.

   (e) Every invertible matrix is equal to a product of elementary matrices.

MATH 320 **Quiz 11** **Name:**___________________________

1. On $\mathbb{R}^n$, define the operations

$$\alpha \oplus \beta = \alpha - \beta$$

$$c \cdot \alpha = -c\alpha$$

Below are listed the vector space axioms for vector addition and scalar multiplication. Next to each axiom, write "yes" if the axiom is satisfied by the above operations and "no" otherwise.

Vector additon:

(a) Associative

(b) Commutative

(c) Identity

(d) Inverses

Scalar multiplication:

(a) Identity

(b) Associative

(c) Distributes over vector addition

(d) Distributes over scalar addition

MATH 320 **Quiz 12** **Name:**____________________________

1. Which of the following sets of vectors $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ in $\mathbb{R}^3$ are subspaces of $\mathbb{R}^3$? Show why or why not.

   (a) All α such that $\alpha_1 \geq 0$.

   (b) All α such that $\alpha_1 + 3\alpha_2 = \alpha_3$.

   (c) All α such that $\alpha_2 = \alpha_1^2$.

   (d) All α such that $\alpha_1 \alpha_2 = 0$.

MATH 320 **Quiz 13** **Name:**____________________________

1. Is the vector $(3, -1, 0, -1)$ in the subspace of $\mathbb{R}^4$ spanned by the vectors $(2, -1, 3, 2)$, $(-1, 1, 1, -3)$, $(1, 1, 9, 5)$?

MATH 320 **Quiz 14** **Name:**________________________

1. Let V be a vector space over F. Given $S \subseteq V$, state the definition of S *is linearly dependent.*

MATH 320 **Quiz 15** **Name:**__________________________

1. Find three vectors in $\mathbb{R}^3$ which are linearly dependent and are such that any two of them are linearly independent.

MATH 320 **Quiz 16** **Name:**________________________

1. Let V be a vector space over a field F. Suppose $\{\alpha, \beta, \gamma\} \subseteq V$ is a linearly independent set of vectors. Prove that $\{\alpha+\beta, \alpha+\gamma, \beta+\gamma\}$ is a linearly independent set of vectors.

MATH 320 **Quiz 17** **Name:**________________________

1. Let V be the vector space over $\mathbb{R}$ of solutions to the homogeneous system of equations:

$$\begin{aligned} x_1 - 2x_2 &= 0 \\ -2x_1 + 4x_2 &= 0 \end{aligned}$$

   What is $dim(V)$?

MATH 320 **Quiz 18** **Name:**____________________________

1. Express $(1, 0, 0)$ as a linear combination of $\alpha_1 = (1, 0, -1)$, $\alpha_2 = (1, 2, 1)$, and $\alpha_3 = (0, -3, 2)$.

MATH 320 **Quiz 19** **Name:**__________________________

1. Let $\mathcal{B} = \{\alpha_1, \alpha_2\}$ be the basis for $\mathbb{R}^2$ where $\alpha_1 = (0, -1)$, $\alpha_2 = (1, 1)$. What are the coordinates of the vector (a, b) with respect to the basis $\mathcal{B}$?

MATH 320 **Quiz 20** **Name:**__________________________

1. Show that the vectors $\alpha_1 = (1, 1, 0, 0)$, $\alpha_2 = (1, 0, 0, 4)$, $\alpha_3 = (0, 0, 1, 1)$, and $\alpha_4 = (0, 0, 0, 2)$ form a basis for $\mathbb{R}^4$.

MATH 320 **Quiz 21** **Name:**____________________________

1. Let $s < n$, and let A be an $s \times n$ matrix with entries in a field F. Prove that there is an non-zero $n \times 1$ column vector X such that $AX = 0$.

MATH 320 **Quiz 22** **Name:**__________________________

1. Let V and W be vector spaces over a field F. Give the definition of a linear transformation from V to W.

MATH 320 **Quiz 23** **Name:**____________________________

1. Describe explicitly the linear transformation T from $\mathbb{R}^2$ to $\mathbb{R}^2$ such that $T((1,0)) = (a,b)$ and $T((0,1)) = (c,d)$.

MATH 320 **Quiz 24** **Name:**____________________

1. Which of the following functions $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation?

   (a) $T(x, y) = (1 + x, y)$
   (b) $T(x, y) = (y, x)$
   (c) $T(x, y) = (x^2, y)$
   (d) $T(x, y) = (\sin x, y)$
   (e) $T(x, y) = (x - y, 0)$

MATH 320 **Quiz 25** **Name:**____________________________

1. Let $T\colon V \to W$ be a linear transformation. Prove that $Range(T) = \{\beta \in W \mid \beta = T(\alpha) \text{ for some } \alpha \in V\}$ is a subspace of W.

MATH 320 **Quiz 26** **Name:**__________________________

1. Describe explicitly a linear transformation $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ such that $Range(T) = Span(\{(1,0,-1),(1,2,2)\}$.

MATH 320 **Quiz 27** **Name:**________________________

1. Let T be the linear transformation $T\colon \mathbb{R}^3 \to \mathbb{R}^2$ defined by

$$T((x_1, x_2, x_3)) = (x_1 + x_2, 2x_3 - x_1)$$

Find the matrix of T relative to the standard basis on $\mathbb{R}^3$ and $\mathbb{R}^2$.

MATH 320 **Quiz 28** **Name:**________________________

1. Let $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by

$$T(x_1, x_2, x_3) = (x_1 + 2x_2 + x_3,\ x_2 + x_3,\ -x_1 + 3x_2 + 4x_3)$$

Find a basis for $Null(T)$.

MATH 320 **Quiz 29** **Name:**__________________________

1. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by

$$T(x_1, x_2) = (-x_2,\ x_1)$$

Find the matrix for T relative to the basis $\{(1,1),\ (1,-1)\}$.

MATH 320 **Quiz 30** **Name:**__________________________

1. Let $T\colon V \to W$ and $S\colon W \to U$ be linear transformations. Prove that $S \circ T\colon V \to U$ is a linear transformation.

MATH 320 **Quiz 31** **Name:**____________________________

1. Let $T\colon V \to W$ be a linear transformation. Prove that T is injective if and only if $Null(T) = \{0\}$.

MATH 320 **Quiz 32** **Name:**____________________________

1. Let $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ be defined by

$$T(x_1, x_2, x_3) = (3x_1,\ x_1 - x_2,\ 2x_1 + x_2 + x_3)$$

Is T invertible?

MATH 320 **Quiz 33** **Name:**__________________________

1. State the definition of an isomorphism between two vector spaces.

MATH 320 **Quiz 34** **Name:**__________________________

1. Compute the determinant of $\begin{pmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{pmatrix}$.

MATH 320 **Quiz 35** **Name:**__________________________

1. Given a linear transformation $T\colon V \to V$, state the definition of an eigenvalue and and eigenvector corresponding to T.

MATH 320 **Quiz 36** **Name:**__________________________

1. Find all eigenvalues for the matrix $\begin{pmatrix} 1 & 2 \\ -2 & -3 \end{pmatrix}$. For each eigenvalue, find the corresponding space of eigenvectors. Is this matrix diagonalizable?