

BASIC PROOF WRITING PROBLEMS
MATH 320
8/25/14

- (1) Let A, B, C , be sets such that $A \subseteq B$, $B \subseteq C$, and $C \subseteq A$. Prove that $A = B = C$.
- (2) Let $f: A \rightarrow B$ and $g: B \rightarrow C$.
- (a) Suppose f and g are both one-to-one. Prove $g \circ f$ is one-to-one.
- (b) Suppose f and g are both onto. Prove $g \circ f$ is onto.
- (c) Give an example where $g \circ f$ is one-to-one, but g is not one-to-one.
- (3) Let $f: A \rightarrow A$, and let f^n denote the composition of f with itself n -times. Suppose that for some n , f^n is the identity function on A , that is $f^n(a) = a$ for all $a \in A$. Prove that f is one-to-one and onto.
- (4) Use induction to prove that $\sum_{i=1}^n i = \frac{n(n+1)}{2}$.
- (5) Given a list of numbers, suppose you can perform moves where you swap the positions of any two consecutive numbers. For example, $(1, 2, 3, 4, 5) \rightarrow (1, 2, 4, 3, 5)$ would be a valid move. Prove that you can always perform a sequence of such moves to get from $(1, 2, \dots, n)$ to $(n, n - 1, \dots, 1)$.