Math 180 Final Review

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This review covers recent material, but the final is comprehensive. Consequently, make sure you go over the two midterm reviews also. Instructions in parenthesis are things you'll be required to do on the test, but probably not explicitly told to do.)

1. Here is a nice problem that reviews a lot of what we've done over the course of the semester.

   (a) Using the formal definition of the derivative, find the derivative of the function $f(x) = x^n$. (No, I don't care that you have it memorized. The point is to show that you can prove that this is really the derivative. Using limit rules is okay here because we're testing your understanding of derivatives, not your understanding of limits.)

   (b) Using your answer to part (a), and the fundamental theorem of calculus, find the integral of the function $g(x) = x^n$. (Make sure that your answer is correct for *all* values of n – is there an exception to the general technique here? If so, how do we handle it?)

   (c) Find the integral of the function $h(x) = x$ two different ways: first, using the definition of the integral by Riemann summation, and second with the geometric characterization that the integral is the area under a curve. Does your answer agree with part (b)?

   (d) Using your answer to the previous part, calculate $\int x^5(x^6 - 7)^3\,dx$ two different ways – once, by multiplying everything out and integrating; and once, by making a u-substitution. (Make sure to clearly write out what u and du are when you make a u-substitution.)

   (e) What is the average value of the function $\ell(x) = x^5(x^6 - 7)^3$ on the interval $[0, 1]$? Given this, what does the Mean Value Theorem for integrals guarantee? (Write in complete sentences and be precise.)

2. Evaluate the following limits. (Clearly label each time you use l'Hôpital's rule. Explain any steps in your calculation that aren't obvious.)

   (a) $\lim_{x\to 0} \dfrac{2\sin(5x)}{\tan(3x)}$

   (b) $\lim_{x\to 0} \dfrac{\ln\cos x}{x\sin x}$

   (c) $\lim_{x\to 0} x\ln x$

   (d) $\lim_{x\to 0} \dfrac{e^x}{x} - \csc x$

3. Consider the given functions and intervals. Do these functions satisfy the hypotheses of the Mean Value Theorem? If so, find *all* points in the interval satisfying the conclusion of the Mean Value theorem; if not, explain which hypotheses are not satisfied, and why.

   (a) $f(x) = 1/x^2$, $[-1, 1]$.

   (b) $g(x) = \sqrt{2x+3}$, $[3, 11]$

4. Sketch a graph of the function $f(x) = x^{2/3}(x-5)$. (Label any asymptotes, local extrema, intercepts, and inflection points. Make sure to explicitly check whether critical points are local minima or maxima, which you could do for example with either the "first-" or "second-derivative test." Be careful to draw each section of the graph with the correct concavity.)

5. Calculate

$$\lim_{n\to\infty} \sum_{k=0}^{n-1} \frac{\pi}{n} \sin\left(\frac{k\pi}{n}\right).$$

using the fundamental theorem of calculus. Express your answer as a single, exact number.

6. Calculate $\displaystyle\int \frac{3x-2}{x^2+2}\,dx$. As a bonus, also calculate $\displaystyle\int \frac{3x-2}{x^2+2x+2}\,dx$.

7. Are the following functions even, odd, or neither?

   (a) $x^3 - \sin x$

   (b) $\sin(x)\cos(x)$

   (c) $\ln(x)$

   (d) $1 - e^{|x|}$

   (e) $\dfrac{1-x^2}{x}$

8. Calculate the derivatives of all six inverse trig functions. (Show all work, including drawing triangles. Again, I don't care that you have these memorized; that's not the point.)